# VALUED KINDS OF KNOWLEDGE AND WAYS OF KNOWING IN MATHEMATICS AND THE TEACHING AND LEARNING OF MATHEMATICS: A WORLDVIEW ANALYSIS

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By

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**Abstract**

This dissertation is a theoretical investigation of the kinds of knowledge and ways of knowing that are valued within mathematics and the teaching and learning of mathematics. Central to this research is a theoretical framework based upon the Traditional Western Worldview and an Indigenous Worldview. This research is based upon a collage of three methodologies: auto/ethnography, Gadamerian hermeneutics, and grounded theory. The initial data source within this dissertation is a personal one, namely the story of how I, the author, have related to mathematics and the teaching and learning of mathematics from my earliest memory up to the moment of genesis for the theoretical worldview used within my research. Analysis of this data gives rise to the collection and analysis of further data, until a theory can be, and is, proposed. In doing this research, I, the author, am most interested in the points of conflict and tension that exist within the different arenas of mathematics and the teaching and learning of mathematics, how these trouble spots relate to the valuing of different kinds of mathematical knowledge and ways of knowing, and ultimately theorizes about how to rectify these problems. In addition to proposing a new theory of mathematics and the teaching and learning of mathematics, this dissertation also proposes a new philosophy of mathematics in support of that theory.

**Acknowledgements**

As is always the case in a monumental undertaking such as this, the acknowledgements are both extensive and worrisome. Who should I include, who will I unintentionally miss? Well, I had better not miss my advisor, Egan Chernoff. As I think back upon the day that I met Egan in person and, tongue in cheek, asked when he was going to start a PhD cohort in mathematics, I never imagined how that question would change my life. Egan has made this experience more than I ever dared to dream it could be, challenging me to push my boundaries (and the boundaries of others) while supporting me however he could so that I could do it. Thank you for encouraging me to "fly my freak flag!" In addition, I wish to acknowledge the support and insights of my committee members, Prof. Bev Brenna, Prof. Bharath Sriraman, Dr. David Burgess, Dr. Debbie Pushor, Dr. Jay Wilson, and Dr. Tim Molnar, as well as my External Examiner, Dr. Lisa Lunney Borden.

Next, I must say thank you to everyone in my story, whether named or not. Neither you, nor I, likely realized at the time that the experiences that I describe in my story would be as impactful as they were on me, and I thank you for making them happen.

I also thank all my friends, family, and students, who likely have heard enough about mathematics, the teaching and learning of mathematics, the Traditional Western worldview, and an Indigenous worldview, to write this dissertation themselves, but really couldn't be bothered. Thank you for all of your "smiling and nodding knowingly," while desperately trying to change the topic of our conversation.

A huge thank you to my two dogs, Euclid and Chevy, who both put up with all my limited amount of attention, but also for making me get out and smell the fresh air while you were smelling everything else. You can look forward to a much-increased frequency in attention and activity!

Finally, and most importantly, I cannot thank enough the many Indigenous knowledge keepers, elders, scholars, and ancestors who openly shared their knowledge and ways of knowing. You have given me so much and have challenged me to strive to make a difference that I believe in. I will never forget this kindness and generosity, and I will continue to strive to use this knowledge for the greater good of all people, the world, and the cosmos.

## Dedication

This dissertation is dedicated to my mother and continuous supporter, Georgina Russell.

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## Abbreviations

**CCF:** Common Curriculum Framework

**NCTM:** National Council of Teachers of Mathematics

**WNCP:** Western and Northern Canadian Protocol

**SIMS:** Second International Mathematics Study: Summary Report for the United States

**Transreform Approach:** The Transreform Approach to mathematics and the teaching and learning of mathematics

## Introduction

Mathematics and I have had a long and, for the most part, positive relationship. As in all relationships, there have been times when I questioned whether I really knew mathematics. At other times I thought I knew it better than anything else. Over the years however, I came to feel that although I loved the math I knew, there was more to it than I had ever been shown. Even as some of the reality of mathematics that I had never known or appreciated started to inch forward into my range of vision, I did not recognize it for what it was or could be – it was just ghost blips on my mathematical radar. It was only when I allowed my thinking about mathematics to interact with my new understandings of two worldviews that I started to realize that the ghost blips were actually parts of mathematics that, for me, had been forced to hide in dark corners, far removed from the mathematics that I knew and loved. This is my exploration of those corners, of turning on the floodlights to expose, consider, and analyze other mathematical possibilities and why they were left behind. This is also a theoretical exploration of the consequences of coming to know and value these other mathematical possibilities not only for myself, but also for students, teachers, society, and even mathematics itself. And so I begin my dissertation where it all began for me, with my story. It is a story of personal experience and academic learnings, it is a story of worldview awareness and mathematical questioning. It is the story of *Mathematics and Me*.

# Mathematics and Me

*Re-Viewing of My Life with Mathematics*

Gale Louise Russell

## Preface: Speaking from Place

**Before beginning the telling of my story**, I first need to clarify a few words that I know hold distinctive meanings for different people. Foundational to my work are the terms First Nations, Métis, Inuit, Aboriginal, and Indigenous, as well as Native (American), and Indian. Like Herman Michell (2005), "I use the term 'Aboriginal' to refer to First Nations, Métis, and Inuit who are recognized under section 35 of the Canadian Constitution (1982)" (p. 33); while at times I am more liberal in my usage, and use Aboriginal to refer to the First Nations, Métis, and Inuit of a specific region of Canada, such as the Aboriginal people of Saskatchewan. This use comes from years of working in the Saskatchewan provincial government, where at least for a time, it was the standard to use Aboriginal to refer to the First Nations and Métis (and Inuit) of Saskatchewan. In this particular case, Inuit is in brackets because Saskatchewan has also held two different positions in this definition of Aboriginal – sometimes included and sometimes not. This "flip-flopping" between definitions is a result of how "membership" in Saskatchewan for Indigenous peoples has been considered. Historically, it is commonly held that Saskatchewan was not home to any Inuit, but over time, more and more Inuit have come to take permanent residence in Saskatchewan, both on and off reserves, so they have come to at times be included in the term Aboriginal in Saskatchewan.

"I use 'First Nations' when referring to the first peoples of whatever region is being discussed" (Herman, 2005, p. 33) – the First Nations of Saskatchewan, the First Nations of Northern Manitoba, the First Nations of Canada. Formally, First Nations is the term used in reference to "the diverse Indigenous peoples in Canada who are connected to 'treaties' and 'reserves' under the Indian Act" (p. 33), and as a result, I typically refrain from using this term when speaking beyond the Canadian border.

Technically, "Métis are persons of mixed blood - European/Aboriginal blood (Indian ancestry); Someone who is distinct from Indian and Inuit, someone who has genealogical ties to Aboriginal ancestry. Note: There is no specified blood quantum" (Canadian Métis Council, n.d.). However, I tend to be less exacting in my use, allowing for Métis to include any people who are part Aboriginal in their heritage and ancestry, regardless of the rest of their ancestral origins.

Most frequently, you will find that I use the term 'Indigenous' to refer to those people who "have a long-term connection, relationship and occupancy of a particular geographical land base" (Herman, 2005, p. 33) – they are the First Peoples of every land. Thus, there are Indigenous people around the world, on every continent, in every region, in every country, in every state, province, and territory.

At this point, a side note is likely required, and that is why I capitalize Aboriginal and Indigenous, when not everyone does. I do this because I see these terms as proper nouns in the English language. They name a specific group (or groups) of people, and as such, as the name of these people, I believe they should be capitalized, as should Saskatoonians, British Columbians, Canadians, Europeans, and Africans. I do so as a sign of respect first and language consistency second.

Returning to the definitions of terms I may use, at times throughout the short history of this country the Aboriginal people of Canada have been referred to (in literature and in public and private spaces) as Savages, Indians, and Natives. Each term has carried significant demeaning connotations as well as, at least in the cases of Indian and Native, been reclaimed as tools of empowerment and decolonization by some (the exploration of which would, in itself, be at least another dissertation). They may appear in my writing as parts of the academic documentation and evidence presented; however, they are, for me, terms that I have not come to understand or value and as such they are not terms that I will choose to use of my own accord.

As in the Canadian context, other geographical and political regions in the world have their own terminology used in reference to Indigenous peoples. For example, in the United States, 'Native Americans' and 'Eskimos' are frequently used terms. Any such terms used in this dissertation will include documentation of the geographical region of the people being discussed.

Finally, specific cultural names rather than broad sweeping (and sometimes contentious) categories such as First Nations, Métis, Inuit, Aboriginal, and Indigenous will be used, whether it is the Maori or the Plains Cree. Just as one would specify Hungarian rather than European if the discussion was specific to Hungarian people – I do this out of respect and to honour the place of the knowledge being shared or story being told.

And now, I believe, the story can begin. It is a story with many branches and many cycles. It is the story of mathematics and me.

**Long before I ever entered school**, I knew that there were some times when story was valued and others when it was not. For instance, when talking to my mom about my friends, story was important, but when I was using numbers, the numbers themselves were all important. It is probably my early awareness of this assumed lack of relationship between story (or context) and mathematics that allowed me to be intrigued by both. In valuing story and mathematics, I did so separately – understanding that the two were like separate islands within a large ocean of knowledge, a Venn diagram with no overlap. The story I now share is the start of how I have come to a new understanding of story and mathematics, an understanding which is aware of a common base on which the two islands of story and mathematics sit, a hidden overlap in that Venn diagram, if you will. This is my story of coming to re-understand and question what mathematics is and what it could be. And, in the way that many stories are told, I go now to the beginning.

**My first mathematical memory** is that of playing *Kingo Bingo* (Sawaka, 1970) with my mother. *Kingo Bingo* was a bingo game held once a week on a local TV station serving as a fundraiser for the local Kinsmen Club.

Every week, my parents and grandparents would buy *Kingo Bingo* cards from the grocery store and give them to me. On the day of the broadcast, Mom and I would move the piano bench in front of the TV and arrange the cards on it with a jar of pennies for markers beside them.

Because of these experiences, by the age of four, I was able to count from 1 to 75 and was able to divide 75 into groups of 15, knowing that there were 5 groups of fifteen in 75. I also learned different shapes – horizontal, vertical, diagonal lines, Xs, Ls, Us, and squares. If asked which column 64 would be in I could answer immediately "under the O". This knowledge of numbers transferred easily to other tasks, such as counting cups of flour when Mom made bread – twenty four. But that soon became too boring – I wanted there to be more than just 24 since I could count higher. I also recall becoming annoyed when my grandfather "miscounted" when we played *Sorry* – after all the most he had to do was to count to 6?! Like Amanda Bean in *Amanda Bean's Amazing Dream* (Woodruff, 1998), I counted "Anything and everything!"

From this vantage point, *Sesame Street* (Connell, 1970) really irritated me as a child. Here it was supposed to be such a great learning show – teaching us letters and numbers, but the most they ever counted was to 20, and that was a rarity. What the Count was counting did not matter – the story was not important to me – we were counting, and we were counting too little for my tastes. I just wanted to learn more numbers!

On the other hand, shows such as *Mr. Dressup* and *The Friendly Giant* held great allure for me – they were all about story and relationships. Numbers were rarely present, and if they did happen to be, they weren't being counted or computed with – they were just part of the story, just like saying that a character was wearing a blue shirt. As a result, I loved these two shows because of their story and the relationships. As an adult, I now realize that *Sesame Street* also had story and relationships, but because of the (my?) emphasis on numbers I did not take note of those other parts, since the stories weren't as important as the mathematics – after all, that was what we were supposed to be learning. Soon, however, *Kingo Bingo, Sesame Street, Mr. Dressup,* and *The Friendly Giant* were replaced by something I had been impatiently waiting for – school.

**When I started school** I loved every class I took – everything was compartmentalized, just as I knew it should be – stories were part of English class and numbers were in mathematics class. To the best of my recollection, there were no stories and no relationships in mathematics classes and there were no numbers (unless mathematically inconsequential) in English – and that made sense to me.

Early on, I also learned there was one right way of doing mathematics – a fact that was made unquestioningly clear to me when, in grade one, I attempted to help the girl behind me do some math. Although, or perhaps because, I was puzzled as to why she did really well in English, but struggled so much in mathematics, I tried explaining to her what we were doing in my own way. Our first grade teacher overheard my explanation and declared that I was not to help anyone – if they needed help, the teacher would help them … the right way. Lesson learned: there is one right way and other wrong ways to do mathematics.

Later that year or the next, as we were working on our addition facts up to 9 + 9, this prior message of doing mathematics the right way came to haunt me. For what seemed like forever, my Mom would practice addition facts with me using a set of flash cards, and I was great at them … except for $8 + 5$. I never could remember that one. Then on one of our practice runs, my mom first showed the $8 + 4$ card: "12!". And then, Mom showed the $8 + 5$ card. I was crushed. I had been doing so well and now I did not know the answer – again! After Mom told me the answer was 13, we were done for that practice run and I reflected on this answer and the previous one. I recall thinking how strange it was that $8 + 5$ was only one more than $8 + 4$, and that 5 was only one more than 4. Now, I knew that this was not the right way of getting the answer to $8 + 5$ because our teacher had not taught it to us, but I was desperate to have a way to know the answer – even if I couldn't tell anyone else how I got it. I would have to lie and say I just knew it if asked. As I continued to get the right answer whenever I thought about these relationships, I started feeling less and less guilty. However, I did continue to feel some level of guilt as it would be more than 20 years before I would share with anyone what I had done to know that $8 + 5 = 13$.

Similarly, in grade three, I was acing my multiplication facts – except $8 \times 7$. No one had told me the mnemonic of $8 \times 7 = 56$ and 5678, although I probably would have mixed it up and thought 8765 so $8 \times 7 = 65$. The cure for my inability happened in the same way for this case as it had for the addition – by consecutive flash cards.

First I was shown the card $8 \times 6$ … "48!", then $8 \times 7$ was shown and I was crushed. After being told, again, that the answer was 56, I thought, "strange, $8 \times 6$ is 48, and if you add 8 to 48 you get 56 which is $8 \times 7$." Thus, just as with $8 + 5$, I had an "illegal" trick to use for $8 \times 7$. From then on, I thought of $8 \times 7$ as 8 more than $8 \times 6$ and I always got the correct answer, even though I was clearly not doing it right. Again, in shame, I hid my secret from my teacher and classmates, with only my success in multiplication overall as giving some relief from my internal humiliation.

By grade four, or perhaps it was grade five, we were on to new and bigger mathematics, such as adding and subtracting decimals, and adding columns of two and three digit numbers. As long as I avoided the "cardinal sin" (as our teacher regularly told us) of not putting the decimal down first before adding or subtracting, I was flying through mathematics. Although I attended church regularly with my parents (the United Church), the notion of a "cardinal" sin was foreign to me; however, I did understand what sin is, so I was very careful to follow this commandment.

Sinning aside, I knew I was doing the best I possibly could, because in mathematics the best is getting 100%, and that is what I was getting. Everything was correct. Then, one evening my father saw me doing some homework involving adding columns of two digit numbers and I explained to him how I was working down the ones column first, adding one's digit r to the sum until I reached the bottom of the list. My dad, interjected and said "the way to add columns of numbers was to build to tens: add the six to the four and the three, five and two together to get 20 with 8 more, that's twenty eight…" I was stymied. What Dad was saying seemed to get the right answer, and it made sense. After all, we had learned that horizontal addition was "commutative", but this was vertical addition and Dad wasn't commuting the entire number, he was not showing his work – he had to be wrong. However, he had piqued my curiousity, so from then on, whenever I had the time, I would check my columns of addition using his method in my head or on a scrap of paper, after using the teacher's method on the paper. Of course, I made sure that any evidence of my using the "wrong" method was erased or thrown out – in a different classroom. I was puzzled and intrigued how very often when the answers did not agree, the answer I had found "the teacher's way" was later marked wrong (and the answer found Dad's way was right). Still, I reminded myself to always do the calculations the teacher's way – the right way.

From late elementary school through to high school, specific memories of mathematics elude me. Perhaps there was nothing to stand out, or perhaps it was because there were too many other things going on: making friends, quarrelling with friends, thinking you are being treated like a kid …. What I do recall is that during this time is that I loved my English classes and their journeys into stories and relationships, and I continued to take delight in mathematics.

In high school, I continued to diligently do my mathematics the right way – the way the teacher taught us. Of course, there were times when I did it the right way, but I made "silly mistakes", like forgetting to put the +/- sign when solving a quadratic equation written as a perfect square:

> *Teacher:* Remember, when you take the square root you have to put in the +/- sign. If the square root sign is already there, just use the sign already given.
>
> *Me:* Oh right (accompanied with a bump of the heel of my hand to my forehead) – how could I have forgotten to do that!

In general, throughout all 12 grades, I was perplexed by the number of students that struggled, and often not succeeding, to get 50%, let alone 100%. I would often over hear them saying to the teacher or friends (including me): "I don't get math," "I don't know what's going on," "I don't know why we are doing this." For me, I had no such dilemma. I got math – just do what you are told to do. All you need to know is what kind of question you are being asked, and you do it because that's how we were told to do it – it's the right way! It all seemed so straightforward and simple. However, the time was coming that I would start to understand their dilemma.

**As it came time to apply for university** (as everyone who had a say in the matter believed I would naturally do) I had a tough choice to make: major in English or major in Mathematics. The hardest part of this decision was that I was going to have to make a commitment to one and leave the other (more or less) behind. In the end, and for reasons I don't even really know, English won out … for the first year. Despite doing equally well in both my first year English and first year mathematics classes, something (again unknown to me) compelled me to change paths and I became a mathematics major.

As second year got under way, the change of majors seemed to pay off in many of my math classes. The probability theory class was really different, but also seemed straightforward, and in linear algebra I was regularly getting that 100% that I so cherished in mathematics.

However, the same was not true with the vector calculus class I was taking. I was not getting 100%, or anything near it. I found myself echoing so many of my classmates and friends from years gone by: "I don't get what to do," "I don't understand this," and "what is this all about." This struggle also happened once in my linear algebra class, but after demonstrating that I could get the right answer a number of different ways, but not in a linear algebra way, my professor conceded and told me what to do (so I could do it the right way).

This resolved my linear algebra problem, and I went back to my frequent marks of 100%; however, I did not find a similar "silver bullet" for my vector calculus course. By the final exam, I was almost completely dejected, believing that I had hit the end of my mathematical road, just as I had seen so many before me do.

On the final, there was a question that involved the intersection of two functions. I recognized the functions by their form – one was the equation of a plane and one of a cylinder. I solved the question completely; however, when I checked my work, I realized that the answer had to be wrong because instead of the circular intersection that I knew it had to be, I had gotten a function that I recognized as being in the form of an ellipse. So, I erased my work and started over – yet again I got an ellipse. I erased and tried again – still an ellipse. I was getting panicked, so I tried a method from another class (I knew I was breaking the right way rule, but I was desperate) – I still got an ellipse. This was how I spent more than an hour of my time for the final exam: erasing, trying again, and erasing again …. For the first time in my life, I had no way (right or wrong) to get what I believed was the right answer, and I was erasing my work, not to hide my cheating, but to hide my failure. In the end I did two things that I had never done before and never thought I would do: gave no answer to a big question and erased my work so much that there was a tear that stretched across half the page. I left the exam defeated – I knew I had failed.

The following day, despite my predictable removal from being in the honours program, and likely from being a mathematics major, perhaps even from being a student at the university, I decided I would check to see if any marks were posted for my other classes. As I made my way into the Department of Mathematics and Statistics from the outside rain to check for postings on my professors' doors, my walk took me past the door of my professor for that vector calculus class. I was relieved to see it was closed, but no sooner had I passed by, when I heard my name coming from behind me.

As I turned, there stood my vector calculus professor, beckoning me to come and talk to her. Walking into her office, my professor took my wet coat and hung it up, and then offered me a cup of tea from a pot of freshly steeped Earl Grey. Sheepishly, I accepted and then we both sat down at her desk. She reached into a file folder and pulled out my final exam and said she wanted to talk to me about it. If I could have turned and run away, I would of, but sitting in the chair, full cup of tea in hand, jacket hanging in a closed closet, and a door that seemed to be miles away, I agreed. She opened my test paper to the ripped page and asked me to explain what had happened there.

I explained that I knew that the one function was a cylinder. She agreed. And the other was a plane. She agreed. I then explained, demonstrating with my pencil and student card that the intersection of the two functions would be a circle, but that I kept getting an ellipse. I just could not get the right answer, so I kept erasing and trying again. In the end, I told her the page was torn and I gave up.

I can still recall the "tsk tsk" sound that my professor made as she slightly pushed the heel of my hand upwards and I saw my student card about to theoretically slice the pencil on a non-right angle. As I visualized my student card slicing through the pencil at that angle my circle disappeared and was replaced by an ellipse. In fact, it was replaced by many ellipses, only one of which was actually a circle. All I could say was "You've got to be kidding!" My professor replied, "No, but now I need to know what you are going to do about it."

As I contemplated and grappled with what this revelation had shown me – that I could regurgitate mathematics, but that there was at least some (possibly a great deal or all) of mathematics that I could regurgitate but didn't really understand – I gradually formulated a reply to her prompt. I explained to my professor that I appeared to have two obvious options: to quit as a mathematics major and go back to being an English major where I was sure I understood things (I was wrong about that too), or else I was going to have to challenge myself and go back to figure out what I really didn't know about the mathematics I thought I knew. I recall thinking, at the time, that it wasn't really a good reply; however, my professor thanked me and said, "that's all I needed to know."

I quickly finished my tea, she brought me my jacket, and without bothering to check if my other marks were posted – it wouldn't matter as I was sure I had failed my vector calculus (I had no doubt then that everything else I did on the exam was wrong as well) – I went back out into the rain. To my surprise, when I received my marks for the term I had not only passed vector calculus, but I had been given the minimum mark in that class that would keep me in the honours program.

I returned shortly after receiving my marks to see my professor again to thank her, at which point she said: "just remember what you said your options were." I have never forgotten them, that in ultimately choosing to stay as a mathematics major I was committing myself to relearning mathematics, to understanding and not just mimicking mathematics. I have never forgotten how on that rainy day that professor, who made me a cup of tea and tilted my hand, started me on a journey of learning to re-think mathematics.

I remained committed to the promise I had made to my vector calculus professor, and I struggled to gain the mathematical understandings I hadn't learned before. Often it was a difficult struggle, with understanding coming months after I had first sought it, or with my receiving of praise (in terms of high marks and recommendations by professors to do a Masters in mathematics) for my outstanding knowledge, knowledge that I guiltily knew, and frequently admitted, was not as spectacular or meaningful as the credit I was being given seemed to indicate. Although I was open and honest about what I felt my deficiencies were, both my professors and fellow students really didn't seem to understand what I was getting at.

Probably the best example of my struggles and of others not understanding them comes from the day that a student, who I had now known for two and a half years and who was in all the same mathematics classes that I was in, came into a classroom of ours to find me clearly in torment over something. Upon his inquiry, I replied that I was finding it very difficult to "wrap my head around" the mathematics we were learning in this particular class. He kindly replied, "like what? I'm getting this class – perhaps I can help you." Seizing upon the opportunity to ask someone who wasn't the professor the "foolish questions" I had (the vast majority of my fellow classmates did not speak English, and I did not speak their languages – other than math, and as noted, I wasn't getting the math), I posed "well, for example, I don't understand what a tensor product is." I was so excited when he replied, "I can help with that!" and then he continued on by giving me, word for word, the definition that was in our textbook.

I thanked him for his answer, but seeing I wasn't yet happy he inquired what was wrong. My reply was one that he did not understand: "I know the definition of a tensor product, or can look it up, but what I want to know is what *is* a tensor product? What does it look like? Does it have hands? If I was to meet a tensor product in a back alley should I shake its hand or run away?" My fellow student was stunned by what I'm sure he thought was pure gibberish that was coming out of my mouth, and he said "All you need to know is…" and he again supplied the definition. Forcing a smile, I said thank you, and made sure to look like his efforts had really helped, but they had not.

When the course was finished, and I was (again) going to check if my marks were posted outside of my professors' doors, I happened to run into the professor for that particular class. He felt obliged to tell me why I had received the second highest mark in the class, and this other student (who had told me what a tensor product was) got the top mark. It was because that student had picked up the language and I hadn't. I replied: "true enough – I actually didn't expect this high of a mark as I really don't understand what we were learning."

Two things related to this incident have stuck with me. First, that professor said, "don't worry about it – later this year in some other class it will all of a sudden come back to you and make sense." I believe, although I never formally tested my knowledge, that he was right. The second point that I remember is not as positive. After we finished our BSc degrees, that same student became a Master's student working with that same professor. After less than four months in, the same professor informed me that his grading system had been faulty as he now realized that just because someone spoke the language didn't mean they understood the concept. I also found out from the student who had tried to help me that he was removed from the Masters' program because he could only speak the language, but did not understand the mathematics. I was saddened to realize that for this particular student (and friend), his time for realizing that he was missing understandings in mathematics did not afford him the opportunity to commit to change and carry on as mine had. Instead, his realization occurred with the firm closing of the door to mathematics that I had not so long ago feared myself.

Another important event occurred in my fourth year when, as a math major, I had a weekly honours seminar at which each of us who were involved were expected to research some area of mathematics (in my case, the mathematics of sounds produced by different drum shapes – music to that point had always been another passion of mine that was also kept separate from mathematics) and present what we had learned to our seminar group. As the classroom we met in was in use prior to our seminar, we frequently met in the department of mathematics and statistics lounge.

One day, while sitting in the lounge before class, one of my fellow students mentioned how his partner, who was an English major,

was seriously contemplating doing her Masters in mathematics. One of the professors who ran our seminar said: "well, that's a bit of leap, isn't it" and I blurted out "not really, after all, mathematics is just another language." It was a thought that had been trying to push its way forward in my thinking for some time as I had worked to relearn mathematics, but had not, to date, ever been said out loud.

As soon as the comment was out of my mouth I braced for the onslaught, but was relieved to see that a number of professors in the lounge decided to take sides on this point and quickly it led to a debate that I was able to observe without involvement. Gradually, the debaters started leaving, but as those of us in the seminar were heading out the door, two professors remained. The last words we heard said from the pair, was the professor of a Christian faith saying to the professor of non-Christian faith: "At least we can agree that God made 0 and 1, and man did the rest," and I heard in my memory my elementary teacher's warning about the cardinal sin of adding and subtracting decimals. I was very happy to get out of the lounge, down the hall to our classroom and be safely sitting at a table, waiting for that week's presentation to begin. I was starting to believe that mathematics and religion were also topics best kept isolated.

After completing my BSc in Mathematics (Honours), and still firmly committed to my relearning of mathematics as I promised my vector calculus professor (much to the, at first, chagrin, and, often later, pleasure of the many university students I tutored in mathematics), I entered into an education program. I had always believed that I wanted to teach, but when I graduated it seemed like half the people from my graduating class of over 600 were going into education, and I had started to doubt whether it was where I should be. However, after my four years studying mathematics, and more importantly relearning and teaching others like me how to relearn mathematics, I knew it was what I wanted to do.

Initially I applied to three university education programs. Two accepted me, sight unseen, merely because I was a mathematics major and they desperately needed mathematics teachers in their parts of Canada. Although the two offers were tempting, I decided that the financial commitment for making such large moves was not something I was willing to take on. The third called me in for an interview at which I was informed that I could not do a secondary education degree with math as a major and English as a minor – I needed to take more physics and biology classes as part of my program so I could be a math major and science minor. I remember noticing how puzzled the advisor I met with was by my combination of major and minors – clearly he felt (or the program he was interviewing me for was designed) as if English and Mathematics do not go together. It never dawned on me at that time that up until just recently this had also been my thinking.

As a result of not wanting to take additional science classes and really wanting to teach both mathematics and English, I decided to apply to the Education program at the same university where I got my BSc degree. This time, there was no issue with my having a mathematics major and English minor as a prospective teacher, although there was one English class that I had to take to fulfill the entrance requirements for an English minor.

Throughout my education classes and internship, I was pleasantly surprised by some of the things that I learned about – such as the use of different concrete models to help students develop understandings of abstract notions in mathematics, such as integers and operations on integers. English was still embedded in story and relationships and mathematics was still abstracted for absolute truth. After 12 months of the education program, I convocated from the College of Education and, too late to get a teaching position for the upcoming school year, I decided to put my name on the substitute teachers list as well as volunteering at my old high school when I had time.

**After eight months of being a substitute teacher and volunteer**, I accepted a full time secondary mathematics teaching position in a small rural school. The student body primarily were from farming families and from a near by First Nations reservation. Although the reservation had its own K-12 school, I was surprised to find out that across the grades K-12 an average of 10% of the students registering in my school in the fall were from the reservation. The impression that I got was that some of the families on the reservation felt that their children would get a better education in a public school, although I never thought to actually inquire into it. Early on, a message concerning the students from the reservation quickly spread through the teaching staff – if we work hard to keep them here until October 1st everything will be great!

I, in my naiveté, thought this meant that if the First Nations students stayed until October 1st, they would most likely stay at our school all year. I knew, from what I had been told in University, that the drop out rate for First Nations students was high in the province, so I was happy to hear that there was something that I could do, for the first month (although, I was prepared to do it for the entire school year) to keep the First Nations students in our school from dropping out. It was years later, when I was elsewhere, that I came to actually understand the message – that if the First Nations students were in our school on October 1st, we would receive all of the funding allocated by the Federal government for each student for the year – regardless of whether the students remained in our school or not. You would think that I would have known something wasn't right given the number of First Nations students who left our school after October 1st to go the school on the reserve every year, but somehow I never connected the dots. I do not know how many of my colleagues knew this truth behind the message that we had all shared – I like to believe none, but I have also never asked.

In each of the grades 10-12 mathematics classes that I taught at that school, I never had more than one First Nations student, and, despite my gut belief that they didn't need to be, the vast majority of those students were most often slated into general mathematics classes rather than the regular academic offerings. Although the general mathematics classes were never intended to be used as modified courses, many schools in the province used registered "struggling" students in these classes and teachers were expected to "just give them the math they really need", without that math ever being defined.

At the end of my third year teaching, a grade 11 student who was First Nations and in my grade 11 Algebra class told me he would not be taking grade 12 mathematics. I pleaded with him to change his mind and to tell me why he was making this decision. His answer that,

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although he finally understood from my classes what it was that mathematics expected of him and that he knew he was capable of doing it, he felt he had missed out so much that it really was not worth his effort or mine. I did my best to try to convince him that my effort was why I had chosen to be in this job, and that it was actually a privilege when I knew I could make a difference in someone's life. He thanked me and told me I had already made a difference in his life. He was not in my grade 12 mathematics class the next year.

It was not until my sixth year of teaching at the school that I had a First Nations student take a grade 12 mathematics course. In order to graduate from grade 12, students were only required to have a grade 11 mathematics credit. Where most of my non-First Nations students were expected by administration, parents, and even themselves to take at least one grade 12 course in mathematics, the same push or motivation did not seem to exist for the First Nations students. In fact, I was often the only voice arguing for the First Nations students I had in grade 11 to go on to take grade 12 mathematics. I could not understand why this was happening and it bothered me a lot.

I regularly reflected on how my students from the reserve struggled with mathematics, wondering what was happening in the prior grades that was causing this chasm in the First Nations students' often they had been in the same prior mathematics classes (as far back as Kindergarten) as the students, who were not First Nations, who did go into my grade 12 mathematics classes. As the only mathematics teacher in the school, and not sure how to broach the subject with my elementary and middle school colleagues who may have taught the students, this reflection remained an internal one – one of those puzzles in teaching that you know should not exist, yet it does, and what to do about it eludes you.

As I was heading towards my seventh year of teaching, I decided that I needed a change. I had found myself falling into routines I did not like – forgetting to consider that I might be unintentionally contributing to the occasional issues that would arise in the classroom or school, and instead placing blame squarely and solely on particular students. I was catching myself wanting to join in in the staff room rants about particular students, and I knew that was not where I wanted to be. So, I applied for and accepted a position across the province from my current school. My new school was larger than the first, although still small, and I was fortunate enough to have a second high school mathematics teacher to work with. With the school being within a larger community, the students came from a (slightly) more diversified family background. Many were from farm families, but others were involved in different forms of industry and commercial business.

The second difference between the two schools is that my new school had no First Nations or Métis students. As a result, I did not have the opportunity at this school to further explore the phenomenon of First Nations students dropping out of mathematics that I had observed at my first school; however, my curiousity and concern in this regard did remain with me.

During my years at both schools, I was fortunate to have the support of my director to, first, be part of a pilot project for a new mathematics curricula at the high school level, and, second, be an implementation leader for those curricula. It was a big change in curricula with respect to how teaching and learning, rather than content, were viewed. I was excited to see the emphasis on the use of hands-on materials and technology in the teaching and learning of mathematics – so much so that I, for the time being, forgot about any questions or concerns I had about the "right way" of mathematics.

Shortly after becoming a pilot teacher I was introduced to a mathematics manipulative called "Alge-tiles®" which could be used to represent different algebraic expressions and equations and to demonstrate what operations on polynomials looked like. Although I knew all about algebra and polynomials, this manipulative was an eye-opener of the kind I had been looking for all those years back when I was struggling with the tensor product. Now, I could see algebra – I could tell if it had hands and I was pretty sure that unless you did not use them properly, polynomials would be friendly. Moreover, the end result from having students use this manipulative was that they would naturally come to do the math the right way! The weekend following my Friday introduction to Alge-tiles®, I picked up construction paper and glue and proceeded to make my own homemade set. I had a plan.

In my grade 10 Algebra class at that time, I had a delightful student who also was my director's daughter. She was bubbly, popular, kind, and smart in everyway except, it seemed, in algebra. Up to this point I had struggled to find ways so that she would understand that x + x does not equal $x^{2,}$ just as apple + apple does not equal house. I later (much later) realized that when she said "Oh, right – I forgot" each time we went over this idea that what I was saying made as much sense as what my grade 10 teacher had told me about the rule for the +/- sign and square roots – we both thought we were being told a rule that just was. There was no sense to it – we just needed to know it – like knowing that 8 + 5 = 13 – just know it!

I arrived at the school early on the Monday morning after my weekend of manipulative making so that I would be sure to catch the student as she walked in the door and invited her to come see me during the lunch hour. Like clockwork, the student arrived with a smile on her face and I introduced her to the Alge-tiles®. Probably because I was so excited about them, the student did not question the validity of the tiles, nor did she seem to feel that such "toys" were beneath her. At the end of her introduction to the tiles, I gave the student a bag full of them and told her that she was allowed to use them anytime she wanted (on tests, on assignments, whenever), and she did. After about a week, I noticed that the bag was opening up less often, and that she was making small drawings, often unrecognizable, along the side of her page, or just pushing the tiles around a bit within the bag. By the end of the month, she brought me the bag of tiles, telling me that she did not need them any more. I took the bag, put it in the top drawer of my desk, and told her they were there for her anytime she wanted them.

To my knowledge, the student never came for the Alge-tiles® again; however, her algebra marks were growing in leaps and bounds, and soon other students were turning to her for help. None of the other students ever wanted to try the tiles, but they did accept this particular student's use of and reference to them, and I started to notice small drawings on the sides of many of their papers too. By the end of the course, the director's daughter had gone from struggling to pass to getting grades in the high 80s and 90s. Of course, I doubted myself (and the Alge-tiles®), thinking that I must have been giving her too many extra partial marks or not checking her work carefully enough.

As an outstanding musician, singer, and dancer, the director's daughter chose to go to a city school to completer her high school education – a school where she could be more deeply involved in the arts, but that also was known for its high academic standards. As her family still lived in the community, she often stopped by for a visit after school hours. I was excited to hear about all of the drama and musicals that she was involved in, but she was even more excited to tell me about her excelling in her mathematics classes. She credited this success to my sharing of the Alge-tiles® with her. As I continued on teaching, this incident also became a point of reoccurring reflection for me; perhaps serving as a counter-balance to the suffocating tension I still felt when I remembered the plight of First Nations students in mathematics that I had previously seeing.

Consequently, I was able to justify the use of the Alge-tiles® as a means to the end – in the end, this student was able to do algebra the "right way." Just like the addition and multiplication tricks I had for a couple of number facts in my elementary years, the Alge-tiles® had merely acted as a springboard to take her to the "right way." After all, she no longer needed them – she knew algebra.

On a different note, as the high school teacher in a small rural school without a guidance councilor, I was accustomed to having students seeking my advice and knowledge about applying for different post-secondary programs: "What mathematics do I have to take," "Do you think I could be a veterinarian," and so on. In that regard, at our first or second pilot teacher meeting, we were told that Math B20, the second of two consecutive grade 11 mathematics courses being developed, was intended to be an entrance requirement for post-secondary programs that did not require students to know about geo-trig or to take a Calculus course. I recall thinking this would be a nice option, particularly for students entering the humanities or fine arts – it would give them an opportunity to take other courses relevant to them in high school.

However, at the next pilot meeting, when the grade 11 (Math A20 and Math B20) curricula were to be shared we were instead given Math 20 and Math A30. Our inquiries into the change led to the explanation that the post-secondary institutions refused to consider a grade 11 credit in mathematics for an entrance requirement. Initially, the curricula writer was planning to start the grade 11 curricula over, but it was suggested to him through an anonymous phone call that instead he rename Math B20 so that it would be a grade 12 course, hence Math A30. I was shocked – weren't entrance requirements based upon what was needed, not name or grade level of the course? Where did this hierarchy of knowledge and eligibility come from, and was it part of mathematics? I was concerned about making the name, rather than the content, the most significant feature of the course, but as always, there is more than enough to keep your mind occupied when you are teaching and I soon moved this information to the back of my memory.

**My experiences as a pilot teacher and implementation leader** for the new curricula also served as a gateway to an unexpected change in my life – that of becoming the first educational consultant of K-12 mathematics at the provincial Ministry of Education. Because of all the experiences I had had working with the new curricula and with teachers in the field, and how much I enjoyed it, this position seemed to have been crafted just for me. In addition, as I was now living in a larger urban setting, other opportunities arose, such as my seeking of a Masters of Education degree. Throughout my almost 12 years in this position I had many experiences that have come to influence how I think about the teaching and learning of mathematics, as did my working on my Masters.

My love of learning and my desire to better understand how mathematics can be taught and learned made continuing on into another degree an obvious pursuit for me. Looking back, I wonder how it was that my interest in the disconnect between First Nations students and mathematics did not become the focus of my studies, but then I have also many stories and experiences between the start of my Masters program and now that are undoubtedly influencing what I remember from that time. It was instead the concept of zero, or more precisely what teachers and students understand about zero that became the focus of my studies. This decision was greatly influenced by my participation on the mathematics team of the Western and Northern Canadian Protocol (WNCP) during the renwal of the *Common Curriculum Frameworks* (CCFs) *for grades k-9* (WNCP, 2006) and *for grades 10-12* (WNCP, 2008) *mathematics*. The CCFs are documents that outline the mathematics content to be taught in each grade in the four western provinces and three territories of Canada (the WNCP member jurisdictions).

Early on in the renewal process, while working on the kindergarten, grade 1, and grade 2 outcomes, I, being a secondary specialist, inquired why students were learning about whole numbers from 1 to 10 and 1 to 100, but never about zero. Those who were elementary specialists sitting around the table appeared to be in shock that such a question was asked: "Zero is too difficult a concept for young children – just look at how late it was developed in history!" I was stymied by the reply, as I did not know the history of zero at that time, so my questioning reply had no real effect: "But if you have a two year old with a pile of Smarties® in front of them and you ask how many they have, they will give you all sorts of numbers from 2 to a zillion, but if you take them away, what two year old doesn't know how many they now have? I'm not suggesting that they learn about limits and infinity – just naming having none of something by its quantity name of 'zero'." In response, it was stated "Over my dead body will these young children be exposed to zero," and the conversation moved on to a different topic.

This conversation, or rather, lack of a conversation, reminded me of many encounters I had had with students in relation to zero: their forgetting to consider it as a number on the number line, ignoring it in a list of data because it was just "nothing," and questioning why everything seemed to be caught up in zero (solving equations, solving inequalities, non-permissible values, test points…). Clearly zero was more than the "trivial case" that it is commonly referred to in higher grades and later courses of mathematics, and I needed to get to know it better – so I did. I started reading about zero, having conversations with others about "nothing," and playing with an empty set. I found out my colleagues from the WNCP mathematics team were right – zero had developed late in history, at least in Western history, and more specifically, Western *written* history. What I learned about zero is a story, or really a set of stories, a few of which I feel are worth sharing as a part of my overall story.

What I found out about zero was both interesting and perplexing. In the written histories of zero, claim to the first use of zero is given to the Mayan culture. During their short existence, the Mayans developed intricate number and mathematical systems, including a complex calendar system that counted backwards to the end of time. Zero played a prominent numerical role in the Mayan society, including each month beginning with day zero, and the inclusion of year zero as well (Barrow, 2000; Kaplan, 1999; Seife, 2000). I found this illuminating, as it had always puzzled me why, when switching from BC to AD we had counted back to one and then started moving forwards again, when it seemed to make sense to count back to zero – like a timer would.

As I read further, I found out that zero appeared, anew, in Babylonia around 400 BC, while the first place value system of numbers was being invented. At that time, a symbol was introduced to represent an empty column on a counting board and even though it was thought of as "a digit, not a number" (Seife, 2000, p. 15) it was the first written representation of a notion of zero known within that part of the world. It seemed odd to me that at that time, zero would not have been recognized as a number, representing a quantity with a measure of none. Instead, Lydon and Kaplan (2000) noted that this new symbol was more like a punctuation mark than a numerical quantity.

Perplexed by this seeming oversight of the Babylonian zero, I continued to read about zero, finding out that through trading and warfare, the Babylonian symbol made its way to other parts of the world, most notably India and Greece (Barrow, 2000; Kaplan, 1999; Seife, 2000). In India, the placeholder symbol was not only accepted, but the notion of a quantity of zero and the symbolic representation of that quantity also came into being. I was actually filled with a sense of relief for zero, it had finally been acknowledged for its quantity (or lack there of). I was also very intrigued to find out that the reason why the Indian mathematicians moved so quickly to the incorporation of the symbol and its numerical meaning was tied to the Hindu religion. There it was – religion playing with mathematics, yet again. Specifically, I found out that because the Hindu religion at that time had numerous deities, all of which represented the balancing of two dichotomous or opposite ideas. In this case, zero, or the void, balanced the notion of the infinite (infinity). (Barrows, 2000; Kaplan, 1999). However, the Indian mathematicians went further than to just name a quantity of nothing with the Babylonian symbol for zero:

The Indian calculators readily defined [zero] to be the result of subtracting any number from itself. In AD 628, the Indian astronomer Brhmagupta defined zero in this way and spelled out the algebraic rules for adding, subtracting, multiplying, and most strikingly of all, dividing with it. (Barrow, 2000, p. 39)

Reading about these developments was almost thrilling, as they quickly led to the start of the base-ten place value system and its related operations that are used today and that I had loved ever since I had started playing bingo with my mother all those years before. Zero and the Indian number system quickly spread to Asia and Arabic countries, as astronomers and bookkeepers happily moved from their use of additive number systems (where the value of a number representation was the sum of all the values shown in the representation, such as used in the Roman Numeral system) when it was realized how much simpler calculations could be (Barrow, 2000; Kaplan, 1999; Seife, 2000).

And so, I began to wonder what all the hubbub had been about zero having being developed so late in history. Sure, the symbol for zero had been developed after those for other quantities, but who really wanted to symbolically represent how much of something they had if they had none of it? Moreover, the 7$^{th}$ century isn't that late in mathematical history! Then, I started to read about Greece's reaction to zero, and it was not good.

Although it is believed that zero officially arrived in Greece during the invasion of the Babylonian Empire in 331 BC, during which they carried "zero off with them, along with women and gold" (Kaplan, 1999, p. 17), it was only in their work on astronomy that zero appeared in Greece at that time. Even within astronomical papyri, zero was only appearing as a placeholder within angle measurements, and not as a numeral for a quantity. In fact, Seife (2000) notes: "the Greeks didn't like zero at all and used it as infrequently as possible. After doing their calculations with Babylonian notation, Greek astronomers usually converted the number back into clunky Greek-style numerals – without zero" (p. 39). Just like how I never told anyone about my tricks to add 8 and 5, multiply 8 and 7, or add columns of numbers quickly, the Greeks were apparently trying to keep their use of zero a secret from everyone else. As I read about their conversions back to the Greek-style (and additive) number system, I saw myself erasing my cheating scribbles in elementary school and my obviously incorrect mathematics during the university exam. I could almost feel their fear of being caught doing their mathematics the wrong way, and the embarrassment of not being able to do it the right way.

But, zero is part of the mathematics I knew and loved – why were the Greeks so standoffish about it? What I found out was that for the Greeks, this new number and numeral carried a lot of old baggage. The first problem went back to the work of the Pythagoreans, a basis of not only Greek mathematics, but also Greek philosophy. "To Pythagoras the connection between shapes and numbers was deep and mystical" (Seife, 2000, p. 27); however, there was no such connection between zero and shapes. For the Pythagoreans, the theories of Pythagoras, and the mystical properties that accompanied those theories were serious business. They were the "right way" to do mathematics and to live life. Accepting zero would put both mathematics and life in jeopardy. Yet again, I was seeing how mathematics and religion could become so entangled that religion would dictate what mathematics did and could be.

The second problem I discovered that the Greek civilization had with zero was philosophical: "if you were a philosopher, you might have had to get a grip on the slippery abstract concept of Nothing and persuade your peers that Nothing could be something at all – not least, something worth studying" (Barrow, 2000, p. 54). Notably, it was this struggle with zero that became a common plaything for authors in the Greek civilization, such as in Homer's story of Odysseus and Cyclops. Being led to believe that Odysseus' name was Noman, Cyclops cries for help were not heeded, as he claimed "Noman is killing me" (Barrow, 2000, p. 60). For, if no man was killing Cyclops, then Cyclops must be mentally ill, and there was nothing (zero) that could be done for that. I enjoyed reading about their use of mathematically referential puns, and started to realize that when I had taught mathematics I too liked to play with words and make mathematical puns. Perhaps mathematics and story were not as isolated as I had always believed them to be.

Finally, I read about how the Greek civilization at the time had problems with a more scientific, or natural philosophy, question about zero: how could a perfect vacuum of empty space exist (a physical representation of nothing)? Pondering such questions naturally led to questioning of religion and the existence of God. In fact, all three of these issues with zero "risked serious disapproval from the religious status quo for letting … thoughts stray into such potentially heretical territory" (Barrow, 2000, p. 54). Thus, zero was, officially through laws, banished from Greece, although Greek astronomers and Italian account keepers often used the Indian zero (and number system) in their "scrap paper" calculations. The Hindu-Arabic numerals (including zero) were even commonly used in the writing of encrypted messages; however, the number system was never allowed to live "above ground," so to speak. As time progressed, other arguments were made against the readmitting of zero into Greek life, including the claim that the Hindu-Arabic numerals, to which zero (0) belongs, were too easy to falsify: "a 0 could be turned into a 6 with a flourish of a pen, for instance" (Seife, 2000, p. 80). It was not until the 13th century, when the mathematician Fibonacci included the Indian zero in his writing, that the government (and hence, by that time, the Catholic church as well) gave in to the powers and pressures of the economic and academic worlds and zero re-immigrated ( officially) to the Greek world and its neighbours. I had never dreamt that zero, or any of mathematics, might be such a religious and philosophical problem.

My research into the history of zero also led me to explore the many roles and unique properties of zero – place holder, quantity, additive identity, multiplicative identity, inverse relationship to infinity (positive, negative, or both) …. It was true, zero is a complex concept. However, I was, at the end of all my reading and discussions even more convinced that the exclusion of zero from the early grades of the curricula was not a well-grounded choice. For the first time ever, I found myself questioning why we teach mathematics topics in the order we do. Zero did not develop late in every culture – the Mayans, Indians, Asians and Arabs all grabbed hold of it immediately and ran. In fact, without zero our number system (base-ten place value) would not exist, so why are we not teaching it right away? Besides, I still could not believe that people in Greece (and the areas under the influence of its knowledge creation) were not aware of a zero quantity just because it had not yet been symbolized. Sticks with notches on it have been considered historically to be evidence of a counting system, but what about sticks without notches – could they not have represented zero of something? Further, I was beginning to question the emphasis placed on symbolic notation as an indication of a concept existing. Sure, it's a lot easier when you can see the symbol for zero to know that people were using a concept, but I could not accept that without symbols the ideas did not already exist. And, there I was, in conflict with *my* beloved mathematics. Mathematics is all about working with symbols – numerals, operations, variables, but I was starting to think that the symbols did not make the math.

This was the first time I started questioning if what I had been taught and experienced about mathematics was all it could be, and it was a very uncomfortable position for me personally. I almost felt like I was betraying mathematics – but what if mathematics had already been betrayed before I came along? In the end, I was fortunate to have had the privilege of being able to make the CCF documents into curricula documents for my home province, and in doing so make changes to outcomes and indicators that I felt necessary to make the curricula fit in my province. I successfully convinced others involved in this process that students needed to be introduced to zero, as a quantity, as soon as they started learning about the whole numbers in Kindergarten.

My Masters' research also supported this decision when grade 2 and 3 students told me that although they did not know what zero was, they definitely knew it was not a number. When I inquired how it was that they were so sure zero was not a number, I was given two answers: if it was, they would have been taught about it when they were taught about 1 to 10 (good point) and if it was, then we would say it when we say the number names (we say twenty and twenty one – if zero was a number we would say twenty-zero – another good point). And so it was that I had made my first public break from the way I had learned to know mathematics – I refused to let history, particularly a version of history that was not representative of everyone's history with mathematics, determine what mathematics would be learned when.

At the same time that I was working on my Masters degree, my work at the Ministry of Education had a major emphasis on the development and delivery of content and pedagogy based workshops to teachers of grades K-12 mathematics throughout the province. Initially, I had a team of 20 teacher leaders to help in the design and delivery of the workshops, but the team soon grew to 83, with 12 different workshops moving from location to location around the province.

Although the Ministry of Education had consultants who focused on issues and initiatives relevant to First Nations and Métis students, many of whom I knew to see working diligently on the same floor that my office was located on, it was through our offering of these 12 workshops around the province that I realized that the drop-out rates of First Nations students in mathematics was not a phenomenon localized to the first school that I taught in, rather it was the norm. However, very few people had even opinions of why, let alone any idea of what might be done differently.

As I started digging into the data collected provincially regarding mathematics, I was shocked to see just how great of a drop-out rate there was for our provincial Aboriginal students, much greater than that of the their non-Aboriginal peers. Something was going dreadfully wrong – there should be no reason for any gap between Aboriginal and non-Aboriginal students getting to grade 12 mathematics, let alone graduating from grade 12. Brain research had shown, time and again, that everyone was mathematically wired.

And so, I began talking to my colleagues who worked explicitly on Aboriginal education. They confirmed that the situation was as bad as I thought it was, if not worse. When I asked why it was happening, my colleagues explained as best as they could that it was because First Nations peoples had different ways of knowing and that the students couldn't see themselves in many subjects, but particularly mathematics and science. I tried to wrap my head around what these different ways of knowing might be – how were they different, and what did it mean to not see yourself in a subject. My colleagues likewise tried to explain this phenomenon, but I still could not understand what they meant by "different ways of knowing." I tried to figure out how I saw myself in mathematics, but the task was bewildering to me – I saw myself doing and knowing mathematics, not being in it. If anything, I was even more confused with the situation than I had been.

One day I found a document, hidden online, that was a collection of lesson ideas for integrating First Nations content into K-5 mathematics, but that document has since "disappeared" into the web ether. I also stumbled onto other documents from other sources that tried to do the same thing; however, they all seemed so trivial to me – counting moccasins and using tipis to study circles. At the time I did not have the active language to describe what I was seeing, but I knew that this was trivializing what it meant to be First Nations. I was sure that there had to be something better to do to help First Nations students in mathematics, but what that might be remained a mystery to me. The thought that mathematics itself might need help never entered my mind – after all, mathematics was a collection of everything that was correct and done in the "right way."

Meanwhile, the statistics continued to tell of the same grim reality for our provincial Aboriginal students, and when I left the Ministry to pursue my Doctorate of Education in 2010, the last publically released Indicators Report provided the following data:

| | Urban | | Rural | | Northern | |
|---|---|---|---|---|---|---|
| | M | F | M | F | M | F |
| Mathematics 10 | | | | | | |
| Non-Aboriginal Students | 68.6 | 73.3 | 71.1 | 76.5 | 58.5 | 59.1 |
| Aboriginal Students | 54.0 | 56.8 | 52.6 | 57.1 | 49.7 | 54.5 |
| Mathematics 20 | | | | | | |
| Non-Aboriginal Students | 67.7 | 71.9 | 69.8 | 75.4 | 63.9 | 66.5 |
| Aboriginal Students | 57.5 | 59.2 | 56.3 | 58.9 | 54.7 | 57.9 |

*Table 1 Mathematics Averages for Non-Aboriginal and Aboriginal Students (Saskatchewan Ministry of Education, 2010, p. 21).*

Not only is the gap between the averages for the Non-Aboriginal and Aboriginal students shown above a concern, but when the difference in drop out rates are considered with this data, the numbers become even more alarming. Lysyk's (2010) first volume of her 2012 report as Provincial Auditor of Saskatchewan expresses concern over the provincial graduation rates, with the data showing that while 72.3 % of all students entering grade 10 in the 2008-09 school year graduated within three years, only 32.7% of the self-declared First Nations, Métis, and Inuit students entering grade 10 graduated within the same time period. The numbers told a disconcerting story all on their own, but for me, when I read the numbers I saw my former First Nations students and their friends, families and communities. The numbers did not tell me their stories resulting from those numbers, nor did they tell me of the stories that might be possible if things were somehow different.

My concern for what was happening to the First Nations, Métis, and Inuit students was intensified when I started to consider the impact of attaining high school mathematics credits, regardless of graduation standing. Two experiences, in particular, brought these concerns to the forefront.

The first was a phone call from a community organization that had advertised a position requiring the successful candidate to plan out the pruning of trees, planting of plants, and general maintenance of plants and lawns on community property. With the upcoming retirement of the current person in charge, an advertisement had been posted and applications were received. Then it was realized that some of the applicants being considered did not have Algebra 30, as stated in the advertisement, because Algebra 30 was no longer a course offered when the applicants had attended high school.

The caller posed the question, "So what math class that is offered now would be the equivalent of Algebra 30?" Having issues with balancing, clumsiness with tools, and an uncanny ability to kill even plants that are supposedly "fool proof", I had no idea what mathematics would be needed by a person employed in such a position, so I asked what from Algebra 30 was needed. The reply was "they need Algebra 30." So, I tried another route and asked what mathematics the person would need to use. The answer was "Algebra 30 – just tell me which course is the same as Algebra 30." Well, none of the new courses were the same as Algebra 30, otherwise the name would not (likely) have been changed. In fact, since the time when Algebra 30 was last offered, mathematics content from different disciplines (algebra, geometry, geo-trig, statistics, probability…) were integrated into a completely different set of classes. In the end, the best I could do was to say that if a student took Math A30 and Math B30, they would have mostly all the content (and then some extra) that someone who had taken Algebra 30 would.

The caller thanked me and ended the call. I sat there for some time after thinking about who I had eliminated from obtaining that position simply by my woefully uninformed answer. Moreover, I wondered whether geo-trig, which is in Math C30 (and not in A30 or B30) would not have been beneficial in such a position (to deal with angles and length measurements) more so than algebra. In the end, I had to just accept that I had given the most truthful and informed answer to the question I could, despite my doubts concerning the validity of the question and thus the impact of my answer.

A second such call I received was from a lady applying for a program at a technical institute in a different province. Her question was, what courses (from the Algebra and Geo-Trig courses offered when she went to school) would be the equivalent of the mathematics entrance requirements for her program. Because the entrance requirements were based upon the WNCP CCF at the time, I was very familiar with the other province's mathematics courses, and after determining that she had obtained both Algebra 30 and Geo-Trig 30 credits in high school (albeit a good number of years earlier), I was able to tell her that she had all the mathematics required (and more). As she gave a big sigh of relief, I then took the opportunity to ask her what program she was applying for. Her response, stenography, was not even in the ballpark of courses I had been thinking about (like computer science, electrical, engineering …). Now, I need to make something perfectly clear – all the mathematics classes in the world would not have made me a good, even an acceptable, stenographer. I had no idea what mathematics a stenographer would need or why, yet I had told this lady that she was "good to go" to enter the program. From my perspective, both of these cases I've just described were using mathematics as a gatekeeper – a hoop to jump through just to prove you could. These experiences added to my concern for First Nations, Métis, and Inuit students struggling with mathematics.

As I tended to do when I encountered something about mathematics that I had not expected, I dug in deeper to see how wide spread the implementation of this kind of mathematical barrier was. I was horrified to discover that it was everywhere – in the public sector, the private sector, and in post-secondary institutions. How had mathematics become this tyrant, judging a person's ability regardless of whether it pertained to the situation? When had it been given such authority and autonomy? With the next renewal of the mathematics curricula underway, I was determined to make sure that mathematics did not become a token barrier to anyone. Surely mathematics was to learn and enjoy, not meant to create a false boundary in one's life.

Meanwhile, the renewal of the WNCP CCFs (2006, 2008) continued on and, following an implementation plan that the publishers of mathematics resources said would be feasible for textbook production, each jurisdiction began an annual release of mathematics curriculum documents starting with kindergarten, grade 1, grade 4, and grade 7 in the first year. Back home in my own province, this process started with the "Saskatchewanizing" of the CCFs – that is integrating the agreed upon content within the CCF documents with the initiatives and foundations for curriculum renewal that were being developed in the province at the same time.

The criteria set out by this Learning Program Renewal required a fair amount of rewording of the CCF documents. The Learning Program Renewal mandated that curricular outcomes target deep understanding. Many of the outcomes in the WNCP CCFs did not meet this criteria, for example, outcome 4 in the grade 4 number strand: "Explain the properties of 0 and 1 for multiplication and the property of 1 for division"(WNCP, 2006, p. 20), which was not only limited in scope, but it also was intrinsically related to the next outcome in grade 4 about the multiplication facts (up to 9 x 9). Within Saskatchewan, these two outcomes were therefore merged into a single outcome, N4.3, in the Grade 4 Mathematics curriculum (Saskatchewan Ministry of Education, 2007).

In addition, in order to convey the level of deep understanding intended by the outcomes in the Saskatchewan curricula, it quickly became evident that the indicators given in the WNCP CCFs tended to read as targeting traditional rote learning, and not deep understanding (see WNCP, 2006, 2008 for examples). This was a result of a consensus decision by the WNCP mathematics team (obtained by pressure from one jurisdiction's legal department) that the indicators (and outcomes) could not contain any terms that suggested how the mathematics could be taught. For example, an indicator could not say "Share and compare…" because the lawyers argued that "share" was telling the teachers how to teach. In the Saskatchewanizating of the CCFs, words like share were allowed, even encouraged, in the indicators to help teachers better understand the kind of teaching strategies being promoted within the new curricula.

A second initiative from the Learning Renewal Program proved more elusive in the renewal of the mathematics curricula – that of infusing the curriculum documents with First Nations and Métis content, perspectives, and ways of knowing. It was at this time that I realized that my colleagues who worked in other subject areas at the ministry were equally unsure of what the First Nations and Métis ways of knowing were. For all of us engaged in this work, it was unclear what might be infused into any curricula to meet this initiative, and what such infusion might look like.

At this point, I feel it's important to note that my preference would have been for such information, ideas, and content to have been put into the original CCF documents; however, one jurisdiction, again, hijacked the consensus process by refusing to allow this to happen. Instead, the other jurisdictions were told that they could add such pieces to what was in the CCF documents when they used the documents to create their own jurisdictional curricula. I did not like the idea of making First Nations and Métis content, perspective and way of knowing an add-on; however, the only other option would to have been to stand my ground at the WNCP table, also refusing to give consensus. I have no doubt that this would have ended the renewal process at the WNCP level, just as I had been told by colleagues that the same kind of issue had ended the initial CCF developments for social studies.

Starting with the first wave of mathematics curricula that were to be renewed and released (kindergarten and grades 1, 4, and 7), those of us working on the infusion mandate for the curricula renewal began by holding a feedback session (formally referred to as "vetting" session) with Elders, First Nations and Métis teachers, and other teachers of First Nations and Métis students. It was a long day of powerful stories from the Elders followed by intense work by everyone in attendance to find places in the outcomes and indicators where they felt First Nations and Métis content, perspectives and ways of knowing could be added in.

At the end of the day, only a few suggestions were made for incorporation, mainly referencing hunting and fishing, but I eagerly took the ideas home and wove them into the outcomes and indicators as suggested. As a part of the vetting process, the suggested incorporations were put into the draft curricula documents, highlighted, and then sent out to the participants for more feedback. The drafts were also shared with the First Nations and Métis Branch of the Ministry of Education for feedback purposes. The responses were unanimous – the additions could have been about anyone, not specifically First Nations and Métis people, and thus they did not serve the purpose intended. Each of the respondents explained, in their own way, that First Nations and Métis content, perspectives and ways of knowing could not be captured in a few words within a single sentence – a mandate of the Learning Program Renewal was that all outcomes and all indicators were each to be one sentence in length. I was told that without the inclusion of story with the cultural meaning of the additions, the additions did not speak of First Nations and Métis content, perspectives, and ways of knowing. Further, such additions would not speak to First Nations and Métis students in a way that they could see themselves being included in the curriculum and hence learning of mathematics. In the end, nothing was "added" as First Nations and Métis content, perspectives or ways of knowing into the kindergarten, grade 1, grade 4, or grade 7 and the curriculum documents went to print "un-infused."

Although there were no curricular changes made, I came to some understandings and thinking that moved me forward personally. In particular, it was that the message that First Nations and Métis content involved story, and the lack of story in mathematics (and curricula) was an issue. I was starting to see that what I had always taken for granted, that mathematics and story were two separate islands, was not the only way that things could be viewed. Other vetting sessions (not focused on the infusion of First Nations and Métis content, perspectives, and ways of knowing) had sometimes raised concerns about the single sentence mandate for outcomes and indicators, but it was in relation to the length (and difficulty) of reading of some of the sentences as well as the desire for more (short and sweet) examples. I must emphasize here that it was not more detail related to an example, not the story of the example, but more examples (such as might be used in a typical test or assignment) that the other sessions had emphasized. What I did not yet understand, nor even know that I needed to understand, was why story was important for at least those involved in the First Nations and Métis vetting session.

In the following year, as we moved on to the grades 2, 5, and 8 mathematics curricula, we again gathered teachers of First Nations and Métis descent to provide insight into what we could incorporate into these three documents to reflect, authentically, First Nations and Métis content, perspectives, and ways of knowing. The day flowed much like the one a year previously, with much enthusiasm and sharing of ideas, but in the end the participants again found it next to impossible to capture meaningful pieces to incorporate into the documents. Having dreaded such an outcome yet again, I had in preparation, considered the question of "if we can't put the stories into the curriculum documents, can we find a way to invite students and teachers to do so?" As the group of teachers spoke to this question, it became clear that it was not any particular story that they were thinking of including, but stories of self, family, and community and how each contributes towards the well being of others. As such, the phrase (and variations there of) “Identify and describe situations relevant to self, family, or community” as incorporated into indicators whenever possible.

I was definitely not within my comfort zone. I loved the idea of story in mathematics, but I also feared it as it broke all the rules of mathematics I had been so successful with – the one “right way,” the one answer, the beauty of abstraction – story called into question these foundational strongholds, the glue of mathematics I knew and loved, but it also brought a sense of integration, of bringing together the two areas I had always loved so much. In contrast to my internal response, the participants in the vetting session felt this was a “good start,” agreeing that it did open the door for teachers and students to bring the stories that were “too big” to fit into the curriculum documents.

In the next (and unfortunately final) year that this type of feedback session occurred (focusing on the grades 3, 6, and 9 mathematics curricula) three things were different from the onset. First, the above-mentioned phrase was already included in an indicator for many of the outcomes in the curriculum drafts supplied. Second, I had brought to share with the group the finalized version of the four goals of K-12 mathematics. And third, there was a unanimous request from the attendees that two new outcomes be included, one for grade 6 and one for grade 9, that specifically focused on Indigenous mathematical content.

The goals of K-12 mathematics that I shared with the session participants were also part of the Learning Program Renewal expectations – each subject area was to identify three or four goals for student learning, to which the outcomes at all grades contributed, that defined what students needed to learn in order to become a 'thinker and doer' within that subject area. Originally in mathematics, the four goals were focused on developing logical thinking, numerical sense, spatial sense, and a positive mathematical attitude. The fourth goal however, had always seemed too limited, almost too naive for what was the real intention of the goal.

As first written, the fourth goal focused on being positive about doing math, persevering, and taking errors in stride; however, what I, the Program Team (an internal group from within the Ministry), and the Reference Committee (a group comprised of representatives from different stakeholder groups such as teachers of mathematics, directors, post-secondary institutions, the Federated Saskatchewan Indian Nations, and so on), envisioned went much deeper and broader than that. Our desire was to have a goal that informed teachers and students that everyone is capable of learning, doing, and using mathematics; that we can learn from how others understand mathematics; and that together we can all move forward in mathematical thinking and doing.

The result of these discussions and reflections on the original goal was the creation of a new goal focused on mathematics as a human endeavor. When I shared the final draft version of the goals with the vetting session attendees, this rewritten goal garnered a great deal of attention, especially from the Elders. They believed that if this goal was a foundation for what happened in mathematics classrooms (which the intention was that it would be) then First Nations and Métis students would be able to see themselves in mathematics and would therefore become more successful in mathematics learning and doing. In particular, they emphasized the importance of valuing where knowledge is from, learning within the context of community, valuing the knowledge others bring to learning, valuing of alternative perspectives and approaches, and the notion that mathematics is developed to meet the needs of situations and determined by the time, place, and people involved. The goal, in final form, follows:

**Mathematics as a Human Endeavour**

> Through their learning of K-12 mathematics, students will develop an understanding of mathematics as a way of knowing the world that all humans are capable of with respect to their personal experiences and needs. Developing an understanding of mathematics as a human endeavour requires students to engage in experiences that:
>
> - value place-based knowledge and learning
> - value learning from and with community
> - encourage and value varying perspectives and approaches to mathematics
> - recognize and value one's evolving strengths and knowledge in learning and doing mathematics
> - recognize and value the strengths and knowledge of others in doing mathematics
> - value and honour reflection and sharing in the construction of mathematical understanding

- recognize errors as stepping stones towards further learning in mathematics
- require self-assessment and goal setting for mathematical learning
- support risk taking (mathematically and personally)
- build self-confidence related to mathematical insights and abilities
- encourage enjoyment, curiosity, and perseverance when encountering new problems
- create appreciation for the many layers, nuances, perspectives, and value of mathematics. (Saskatchewan Ministry of Education, 2009a, pp.9-10)

This goal, along with the logical thinking goal, is shown in the curriculum documents to be foundational to every outcome, that is, in learning towards every outcome, K-12, students should be having experiences that help them to attain this overarching goal of understanding (and taking part in) mathematics as a human endeavour. The Elders' positive responses to this goal, and its relationship to all the outcomes we had been struggling to infuse with First Nations and Métis content, perspectives, and ways of knowing, were very rewarding to me, as I felt that, perhaps, I was finally starting to get a glimpse into understanding the divide that I had seen (and read about) so often between First Nations and Métis students and mathematics. Moreover, the divide need not exist. That understanding however, was starting to put increasing pressures upon what I had always known (and believed) mathematics to be.

The request for the inclusion of First Nations and Métis mathematics outcomes in the grades 6, and 9 curricula resulted in my writing two new (outcomes not found within the WNCP CCFs), one that considered the role and use of quantity in Indigenous cultures and communities and the other focusing on chance and data within Indigenous cultures and communities. Ultimately, the outcomes were included in the grades 6 and 9 curricula documents (respectively), with "Indigenous" being changed to "First Nations and Métis" to make the outcomes read as more reflective of Canada and of the province of Saskatchewan. These outcomes have created a fair amount of discussion amongst teachers and consultants; however, the level to which they have been implemented throughout the province is unknown.

I also took from this part of our discussions the message that Indigenous peoples did have and use mathematics, but that the mathematics often looked different, while still achieving similar goals. And there it was – my first outright encounter with a challenge to the notion that there was one "right way" to do mathematics since my elementary school years. It was thrilling to know that this challenge was being made, but at that time, I still did not know the extent to which I would eventually come to challenge the authority of the mathematics I loved so dearly.

Coinciding nicely, almost prophetically, with the challenge of writing the two outcomes about First Nations and Métis use, representation, and knowledge of quantity and probability was by my attendance at a mathematics education conference. Although I attended many interesting presentations and was part of a working group looking at the use of visualization in the teaching and learning of mathematics, there is one moment from my time at this conference that trumps all else, that of meeting Louise Poirier. I met Louise when she happened to sit down beside me at the banquet for the conference, and I knew instantly that she had a story she was burning to tell, and I just knew that I wanted to hear it.

Louise had just returned from a trip to Northern Quebec where she was starting to work with Elders in the Inuit communities on developing ways to talk about mathematics in their native language (Inuktitut), and in particular, how to speak about the base-ten number system. I was astonished that they did not already have such terminology in their language, and I asked Louise how this had "slipped under the radar" for so long.

She explained how it had recently been legally mandated that all students in Inuit communities in Northern Quebec were to be taught in Inuktitut for the first three years of school. The first students to go through the three years of schooling in Inuktitut had just moved on to grade four and they were (outstandingly) out-performing everyone's expectations in every subject – except mathematics. Upon looking into the situation (thankfully no one believed it was because Inuit people just can't do math!), the source of the problem soon emerged. The students had, for three years, been learning about numbers and the four operations using the Inuit number system, which is base 20 with a sub-base of 5. What this means is that in their native language the Inuit of Quebec do not have a place values that are powers of 10 (1, 10, 100…). Instead, their number system counts by powers of 20 (1, 20, 400…). Thus, where, in base ten, we would describe 124 as being 4 ones, 2 tens, and 1 one hundred, the Inuit of Quebec would describe a quantity of 124 as being made up of 4 ones and 6 twenties. But actually, that's not even correct because they subgroup their place values into groups of 5, and 6 twenties is 5 groups of 20 and 1 group of 20, so 124 would be described as 4 ones, 1 twenty and 1 group of 5 twenties. Further, in Inuktitut, higher place values are described first, and so 124 is literally said as 1 group of 5 twenties, and 1 twenty, and 4 ones.

To say my socks were blown off is a huge understatement! I had never before heard of anyone in the present day using any number system other than base 10 (except for the occasional use of Roman Numerals by clock and film makers). How could this be the case? Louise told me that the answer partly lay in the fact that the teachers for this new initiative first and foremost had to speak Inuktitut, which meant that many were not "academically" trained teachers, and they taught the students the mathematics of their language. The second reason was that never before had the Inuit people of Northern Quebec had Western schooling delivered in Inukitut – they had always learned in either English or French as decided upon by the individual communities (or as decided for them). Therefore, no one had ever had a need to work with base-ten mathematics in Inuktitut.

The difference in number system bases was not the only reason that the students in grade four "couldn't do math," there was still more that Louise had to share regarding their base 20 (sub-base 5) number system. Another part of history of the number system of the Inuit in northern Quebec was that until colonization their language was strictly oral and so the students had learned the traditional ways of calculating in Inuktitut – orally and in their heads. What was eventually realized was that these Inuit students were not only really good at base 20 (sub-base 5) math – they did it all in their heads – no paper and pencil, no written algorithms.

But Louise wasn't done her story! A final complication in the work that she was just starting with the elders was that, within Inuktitut, numbers were not considered in isolation or abstraction. Rather, numbers were part of the story of a context or place, and as such, there are up to six different words for each number that were are for different types of contexts. As an example, Louise gave me a sheet of paper with six squares drawn on it, and in each square were an image and a (very long) word written below it. The images showed the six contexts for the quantity three and the word used for each: three in something, three of something, groups of three, the digit (numeral) three, a playing card that is a 3, and the arrangement of dots for three on a die. The first of the three words and images were from pre-colonial Inuktitut, while the other three words and images originated from post-colonial contexts.
Consequently, when talking about numbers or doing calculations (in their heads), the Inuit students had to be careful to use the correct word so that the situation made sense. However, very little (if any) of the mathematics that they would have been tested on (in English or French) in grade four would have been in context and the emphasis was undoubtedly on symbolic computations designed to test if the students "knew their math." The grade 4 students knew their math, but the test designers did not know the students' language, math, or history. I still have the original copy that Louise gave me on that day (you can find a copy in Poirier, 2007) – I kept it because of how much it impacted me at the time – little did I know just how far that impact would reach (into this

dissertation and beyond).

Shortly after my return from this conference, as I was working on the renewal of the mathematics curricula, I sought out additional assistance from Dr. Edward Doolittle, a mathematician at First Nations University of Canada. I had asked Edward to help me edit the renewed documents and to provide any grammatical, mathematical, or First Nations and Métis content, perspectives, and ways of knowing suggestions that he might have.

As I too was reading over the documents, I pondered out loud if a particular indicator might be too difficult for the students in that grade. It wasn't the mathematical idea behind the indicator that I was wondering about, but rather the language I had used in writing the indicator. When Edward read over the indicator, one about having students explain with examples what the word equal meant to them, he told me that he wasn't sure if it was too difficult for the grade, but that it was very important for First Nations and Métis students. Caught off guard by the second part of his reply, I inquired why it was important for those students. He explained that in his experience, in many First Nations and Métis communities, the English word equal was used to mean "fair or for the good of the community."

For years I had been hearing from my French colleagues at the ministry that translating from English to French (or vice versa) was never a one-to-one correspondence. I thought, with my limited French language knowledge, I understood what they meant, but Edward's comment took me to a far deeper level of thinking about translation. It wasn't just a case of "we don't say it that way, we say it this way", it was a case of the word carrying a completely different, culturally and place-based, meaning. I later learned that this was called polysemy (a word having multiple meanings).

Of course, I knew many examples of polysemy, such as "grave" – it can be where someone is buried, but it also can be that person's serious mood or expression. The case of the polysemy of "equal" however, was fundamentally different, at least from my perspective. What Edward had made me aware of was not just an alternate meaning – it was a completely different view of the same general notion. It had to have been a consequence of colonization, of the forced translation between two languages, which had resulted in this difference in meaning, yet within this process of translation the original meaning managed to survive within the new word in the new language.

In my mind's eye, I imagined being a First Nations person (or more likely an English speaking person making decisions for the First Nations), finding the closest translation for the First Nations word for "fair or for the good of the community" to be this English word "equal," and despite them not being exactly the same it "would have to do." I thought about how in English we have both equality and equity, and I was wondering why the translation had not been to the word equity. However, I then thought about how "equity", although a word I had known for some time, was not one did not seem to come into regular use until the recent years. The notion of making things equitable for different people was something that had only recently come to be de rigueur in my experience within Western society.

Curious if this was nothing more than just whimsical speculation on my part, I looked into the etymology of the words equal and equity and found the following in the Online Etymology Dictionary (2015):

**equity (n)**

early 14c., "quality of being equal or fair, impartiality in dealing with others," from Old French *equite* (13c.), from Latin*aequitatem* (nominative *aequitas*) "equality, uniformity, conformity, symmetry; fairness, equal rights; kindness, moderation," from *aequus* "even, just, equal" (see **equal** (adj.)).

With colonization, and specifically the translation between English and the First Nations languages (I cannot speak to translations between French and First Nations languages), even if equity was being used by English speakers, there was a good chance that equity and equality were being used interchangeably to imply sameness (just like in mathematics), so there would have been no reason to pick "equity" over "equality." I found these nuances about the words "equal" and "equity" intriguing to say the least – how was it that these two different languages ended up emphasizing a different idea in their language – sameness or fairness?

From a mathematical perspective, on the other hand, I had other concerns. What would mathematics look like to a student who carries the understanding of equal meaning "fair or for the good of the community" instead of "same?" It wasn't that I wanted to go running into all such First Nations and Métis communities screaming "equal means the same," I actually like the fact that this difference exists. But what if teachers don't know that this is happening and therefore don't know to recognize, value, and celebrate this alternative meaning while also helping the students understand equal within a mathematics context? And, what other (what I have come to term) culturally polysemic words are out there that need to be recognized, valued, celebrated, and possibly given a different context within mathematics teaching and learning? Initially, this was to be my dissertation, but as my experiences continued, it became obvious that there were many other questions that I needed to answer, or at least grapple and become comfortable with, first.

My experiences with the renewal of the mathematics curricula (and all others that were related to it) had stoked the embers of my curiousity, rekindling my desire to think deeper and more broadly about mathematics – what it is, and its relationship to schooling – for all students, including First Nations and Métis.

Synchronizing, perhaps perfectly, with the renewal, other events in my life contributed to my desire to pursue these ideas with more ardour. There was my introduction to the field of ethnomathematics (a contentious word, but which for now I will define as mathematics as found within and defined by culture), which introduced me to even more examples of mathematical thinking and approaches foreign to my own experiences. Within my inauguration into the field of ethnomathematics also came my first concrete yet academically documented introduction to colonization and the impacts it has had on Indigenous people around the world. It was as ethnomathematics and colonization moved to the forefront of my thinking that I embarked upon the journey that this writing is seeking to bring to culmination – my PhD dissertation. However, before I can get to the end, I must continue from where I left off, which is the start of my PhD studies.

**In the first term of my PhD program**, I took two courses, Trends and Issues in Mathematics Education and Decolonizing Aboriginal Education. Whereas the Trends and Issues in Mathematics Education was in many ways a confirming and consoling course for me (allowing me the space to explore my at least self-controversial thoughts about mathematics), Decolonizing Aboriginal Education was like a tornado of new ideas, new understandings, and new challenges.

The Trends and Issues in Mathematics Education course confirmed for me, academically, that there was something legitimate about the way I had, for a number of years, been coming to question, at least internally, what mathematics was and could be. In that way, the class helped to, if not alleviate, then lesson the severity of my nervousness to pursue thinking about mathematics differently. In giving me that comfort and security, this class allowed me to formally verbalize my thoughts in discussions with others (I do beg forgiveness for those who perhaps I pushed too hard to come to my way of thinking and questioning – like a bully of mathematical reform – and I thank them for pushing back and ultimately encouraging me to better ground and fortify my thinking).

The Decolonizing Aboriginal Education course, on the other hand, continuously overwhelmed me with ideas and understandings related to colonization and its (continuing) impacts upon Indigenous peoples, and ultimately non-Indigenous peoples as well. Although internally I had always found it odd that my relatives had just been given land to homestead on and set up communities, I had not been aware of the cost of that kindness upon the first peoples of this land. I was starting to understand the great privilege I had been born into and its cost to others.

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Reflecting upon the teaching and learning of mathematics in this light, and considering the things and experiences I had learned over the past 10 or more years, I was now starting to wonder if my beloved mathematics wasn't part of the mechanisms of colonization. Far beyond its being a gate keeper, was it also the source of an outright denial of anything that an Indigenous person, being newly introduced to the Western word "mathematics" might want to associate with it? And then I started our assigned reading for the next week, Leroy Little Bear's (2000) *Jagged Worldviews Colliding* – almost instantly the margins of every page were filling up with notes, notes that said "this is what mathematics is missing" and "this is what mathematics should be", and then later on "this is what the mathematics in schools is" and "this is why mathematics is not working for so many students." Finally, I had the (start) of a framework that might give me insight (and possible solutions, if I was lucky) into all of my amassing questions about mathematics, the teaching and learning of mathematics, First Nations and Métis students, and the relationships between the three.

And so ends the story of mathematics and me, at least up to this point in time. What now might be the continuation of this story is not yet far enough removed from myself to share. My story will continue, but for now my research needs to take center stage, starting with a more in-depth study of the two worldviews introduced to me in Little Bear's (2000) *Jagged Worldviews Colliding.*

## Contrasting Worldviews and the Emergence of a Theoretical Framework

As noted at the end of my story, Little Bear's (2000) description of the two different worldviews, which he and other Indigenous people often find themselves both trapped within and torn apart by, was an eye-opening and soul stirring event in my life. Finally, I had found someone who could speak to me about First Nations ways of knowing, and what they said resonated with the many twists, turns, conflicts, and ultimately philosophical changes that I had experienced with respect to my thinking about mathematics and the teaching and learning of mathematics over the years. His descriptions called to me both intellectually and emotionally, and I knew it was a call I needed to immerse myself within.

As I researched and explored these two worldviews, I also started to see a foundation, a theoretical framework, emerging that might be useful in both theoretical and empirical research. Through the sharing of my ideas about this framework in discussion groups, workshops, and at conference presentations; however, I realized that there were a number of understandings about worldviews that I was assuming, but not explicitly revealing. Having learned from these experiences, I first clarify what I mean by worldview and what the name given to a worldview does and does not imply in my thinking and research before elucidating the theoretical worldview framework that is central to this dissertation.

### An Introduction to Worldviews

In the broadest sense, one's 'worldview' is how one views and is in the world: "*Worldview* may be defined simply as the way people think about themselves, their environments, and abstract ideas such as truth, beauty, causality, time, and space" (Allen & Crawley, 1998, p. 113). Worldviews are how we make sense of the world around us and how we function on our own and with others. However, Shelbert (2003) explains that worldviews are structures that are often unknown to the holders of them:

> They are taken for granted as the roads to be traveled and often escape the processes of questioning. Thus the realm of worldview structures is neither sacred practice nor embraced doctrine of what is true and normative, but the frame, the pattern, the paradigm that shapes understanding. (p. 62)

Yet, "worldviews are important precisely because they provide us with an overall meaningful perspective about life and the world in which we live" (Irzik & Nola, 2007, p. 95). Often, worldviews become apparent first to outsiders grounded within a different worldview when they begin to question why other groups of people make decisions and act in the ways they do.

Many different worldviews have been documented, researched, and classified. For example, there are different religious worldviews, political worldviews, cultural worldviews, and economic worldviews, to name just a few categories in research and literature. In each instance, at the heart of each of these identified worldviews are guiding principles based upon which holders of those worldviews are said to be in the world, and those principles set foundations with respect to the category (categories) that the worldview might classified as. For instance, religious worldviews, which generally speaking focus upon questions such as "Who am I?", "Where do I come from?", "What is my purpose in life", provide foundations about how one is to live one's life, what to believe about the origin of one's self, community, and the world, as well as for how to analyze and judge alternative or new information related to these questions. Likewise, an economic worldview presents a foundation of how to analyze, judge, and implement alternative or new information related to economic theories, decisions, and undertakings.

In essence, "social practices promote the worldviews; the worldviews dictate the appropriate thought processes; and the thought processes both justify the worldviews and support the social practices" (Nisbett, 2003, Introduction section, para. 18). Considering the two examples of religious and economic worldviews, it can be seen that a worldview sets guidelines for ascertaining the value of different kinds of knowledge and different ways of coming to that knowledge. In this way, broader than these two examples, yet more specific than "how one views and is in the world", *worldviews delineate how to determine what knowledge and ways of knowing are of value to the holder of the worldview. A worldview is like a lens through which we look at and seek knowledge, a lens that filters in very specific ways, what we see and where we go looking.*

"Worldviews", as I have come to know, understand, and work with the term, are also sometimes referred to in different literature sources as paradigms, epistemologies, beliefs, or belief systems; however, I must emphasize that these terms are not always interchangeable. In some contexts, these alternate words are used in ways which are quite different from that which I present here (and quite rightfully so in my thinking), so it is important to note that where I might reference work that uses these alternate terms, it should not be assumed that I would consider all references using these terms, or even worldview, as relevant to my work that is to follow.

Although there are many (I think 'an infinite number of' would even be appropriate here) different worldviews, my reading of *Jagged Worldviews Colliding* (Little Bear, 2000) introduced me to two specific worldviews, which have ultimately come to be the foundation of the theoretical framework I will soon describe in detail. Feeling the academic need to name these two worldviews, I have come to call them *the Traditional Western* worldview and *an Indigenous* worldview, and the reason for these names, including the article introducing each of them, will be explained when the nature of these worldviews are discussed in the upcoming sections. Right now however, I must explain that first, I acknowledge that in naming the worldviews my actions can be viewed as firmly placing the kind of knowledge I am valuing within that of the Traditional Western worldview. Second, in reference to the terminology noted above, the two terms I have assigned to the worldviews are not standardized in the literature. Each of these worldviews is described, and sometimes named using different but inconsistent terminology, but it is only within Little Bear's (2000) work that I have found a comprehensive study of the two worldviews presented together. Further, in that work no "names" are every officially given to either of the worldviews. Instead, in what one might describe as being grounded within an Indigenous worldview, Little Bear resists the Western worldview temptation to classify, choosing instead to be at all times descriptive. I on the other hand have given in to this temptation, not to take credit for the identification and explanation of the two worldviews, but in attempt to present my research in what I believe will be seen as a scholarly way. However, I also wish to note that on one front of Western worldview temptations, I am choosing to resist, and will not be using acronyms for the two worldviews. This is the only place where the reader will find IW or TWW in this document.

In so naming these two worldviews, it is essential to qualify that "the qualities identified for both Indigenous and Western systems represent tendencies rather than fixed traits, and thus must be used cautiously to avoid overgeneralization" (Barnhart & Kawagley, 2005, p. 11). Therefore, although the descriptions to be given of the knowledge and ways of knowing that are of value within an Indigenous worldview and the Traditional Western worldview are being linked to two cultures via their names (Indigenous and Western, respectively), it is paramount to not misconstrue one's membership within one of these two particular cultures to imply one's worldview as well. An Indigenous person need not hold an Indigenous worldview, nor a person of Western origins hold the Traditional Western worldview (or any other specific or general

Western worldview). Likewise, someone grounded within the Traditional Western worldview need not be of Western descent, nor someone grounded within an Indigenous worldview need be of Indigenous descent. These names for the worldviews do not indicate membership; rather, they reflect what one or more individuals have noticed (what was believed to be a trend) within a particular group (cultural, religious, political, socio-economical…). Further, these two worldviews should not be held as the only worldviews that might be associated with either Western or Indigenous cultures. There are very likely other Western worldviews (although a general Western worldview would seem unlikely) as well as worldviews specific to particular indigenous peoples.

With the above caveat given regarding worldview membership, “it has become infinitely clear that you will notice differently if you come from a distinct cultural background” (Meyer, 2003a, p. 251) because of cultural differences and nuances. Meyer further explains that “how one experiences the environment plays a huge role in how the world is understood and defined” (Meyer, 1998 p. 23). It is thus no surprise that “Differences between Western and indigenous conceptions of the world have always provided stark contrasts” (Smith, 1999, p. 43). It is within these differences that I contend, and hope the reader will find, the strength of this theoretical framework lies.

And so, as I head into my re-presenting (as I am merely pulling together the work of many different researchers and theorists) of my understanding of two worldviews, I am reminded by Ermine (1995) that

> The year 1492 marked the first meeting of two separate world-views, each on its own uncharted course of exploration and discovery for purposeful knowledge. The encounter featured two diametric trajectories into the realm of knowledge. One was bound for an uncharted destination in outer space, the physical, and the other was on a delicate path into inner space, the metaphysical. (p. 101)

The following discussion of the theoretical framework, and more specifically the two worldviews that the framework is built upon, takes us along both trajectories that Ermine describes, each of which will eventually help to illuminate themes or concepts underlying my story of mathematics and the teaching and learning of mathematics. And, in the end, I look forward to the prospect that this framework may suggest a way to remove or re-envision the tensions and conflicts I experienced and still experience in relation to mathematics and the teaching and learning of mathematics.

And so, this excursion into the two worldviews begins with the Traditional Western Worldview as I have come to understand it (through the works of Absolon, &, 2005; Ermine, 1995; Kovach, 2009; Little Bear, 2000; Malcolm, Sutherland, & Keane, 2008; Matthews, 2011; Meyer, 2003b; Michell, 2005; Nisbett, 2003; Roy & Morgan, 2008; Shelbert, 2003; Snively, & Coriglia, 2001; Sternberg, Barrett, Blood, Glanfield, Lunney Borden, McDonnell, Nicol, and Weston, 2010; Strega, 2005; Van Eijck, & Roth, 2007). Although I understand that "a worldview should not be judged with respect to another, because any worldview is transcendental by nature" (Kawasaki, 2006, p. 43), I also acknowledge that the understandings I have developed as I explored what these worldviews are about – what knowledge and ways of knowing each values – is necessarily influenced and biased by my own worldview. So too, I acknowledge that the works I have read and engaged with may also be afflicted by this same condition, as the writers try to capture their (inevitably self-biased) understandings within words. It will be this way when trying to understand any worldview; however, the openness (level of rigidity) of the worldview will, in and of itself, define what kinds of bias may actually enter in. One might then think that exploring worldviews in any circumstance is pointless, but I hold, based upon my personal experiences, that if all one gains out of the process is a better understanding of how one views and is in the world, the exercise is, for no other reason, worth it.

There is one more concern that, entering into worldview explorations, discussions, and ultimately the using them as tools in research analysis, that I would be remiss in not reflecting upon at this point, and that is the concern of cultural appropriation. As a Western woman and person (although not subscribing to the Traditional Western worldview), I need to be very cautious in how I portray an Indigenous worldview and the claims that I make in reference to it, because I cannot claim to have any Indigenous knowledges that would necessarily ensure my enactment of an Indigenous worldview in a truly Indigenous way. All I can do is to understand and use an Indigenous worldview as my own worldview allows me to. Having stated that, I have informally received confirmation from various Indigenous elders, knowledge keepers, and others that the work I am doing is "good work" and that its importance and significance should not be underestimated. It is with those reassurances that I continue to move forward in my research.

### The Traditional Western Worldview

There are many different worldviews that can be related to Western culture(s) and by choosing to use the name *the Traditional Western* worldview I am attempting to highlight that what I am about to describe is a very particular and distinctive worldview amongst all possible Western worldviews. It is a singular and unique worldview that is common within Western culture, which is strongly grounded within notions stemming from Greek and other European history, such as atomism, positivism, logical reasoning, and Descartes' philosophy of man. For this reason, I intentionally call this worldview *the* Traditional worldview as opposed to *a* Traditional worldview, as I have come to believe that it is unique amongst other Western worldviews, along with being very distinct from an Indigenous worldview.

In the broadest sense, it can be said that the knowledge and ways of knowing valued by the Traditional Western worldview are grounded within separation, decontextualization, and abstraction. (Ermine, 1995; Little Bear, 2000; Malcolm et. al., 2008; Meyer, 2003b; Michell, 2005; Nisbett, 2003; Roy & Morgan, 2008; Shelbert, 2003). These fundamental foundations of this worldview give rise to distinct values that are very specific and definite about what are acceptable kinds of knowledge and ways of knowing.

The Traditional Western worldview upholds "the value systems of Western Europeans [which are] linear and singular, static and objective" (Little Bear, 2000, p. 82). Further, the Traditional Western worldview "rests on a dualistic foundation, in which quantities such as rationality, reason, objectivity and impartiality are privileged over and opposed to irrationality, emotion, subjectivity, and partiality" (Strega, 2005, p. 203). The Traditional Western worldview "*presupposes* certain metaphysical and procedural or methodological commitments: first the existence of an external world that is independent of the observer; second the universality of causation in that world … and third the constancy of causation" (Matthews, 2011, p. 6). It is upon these assumptions that the more specific characteristics of the Traditional Western worldview are built.

**Absolute truth.**

Within the Traditional Western worldview, absolute truth is the kind of knowledge that is sought and valued. This absolute truth is believed to be possible because the world is viewed as "objects – discrete and unconnected *things*" (Nisbett, 2003, Holism vs Analysis section, para. 7), therefore "keeping everything separate from ourselves" (Ermine, 1995, pp. 102-103) renders ideas about the objects necessarily true or false. Thus, by distancing the knower from the object to be known, and by separating the object from all other objects, the Traditional Western worldview holds that the truth of that object, the universal knowledge of that object, can be discovered.

**Compartmentalization, categorization, and isolation = hierarchies and abstraction.**

The foundational beliefs about Nature within the Traditional Western worldview are essential to the search for and upholding of its truths. Specifically,

> Nature is viewed as a threefold entity: the world of inanimate forces of wind and rock and earth, the world of organic life, from micro-organisms such as viruses and bacteria to grasses, flowers, trees, and plants; the world of animals, those beings with *amima,* yet not a soul, and radically positioned below humans. In contrast to this tripartite 'nature,' humans, it is claimed are radically different: they are animate beings endowed with reason, with self-consciousness, and with genuine decision-making power. (Shelbert, 2003, p. 62)

This compartmentalization of Nature, with humans positioned above all else, results in an assumed hierarchy within Nature and amongst humans. Ultimately, "the natural world is divided up, and that nature serves as a backdrop to human society" (Roy & Morgan, 2008, p. 237). With hierarchies being established amongst the objects that are to be known and amongst the knowers of those objects, the natural consequence is that the knowledge itself is also viewed as hierarchical (Little Bear, 2000). Consequently, because of the hierarchies of knowledge and knowers, the Traditional Western worldview is most "concerned with personal goals of self-aggrandizement" (Nisbett, 2003, The Non-Western Self section, para. 1), that is, moving up the "ladder" of hierarchy of humans happens by moving up the "ladder" of hierarchy of knowledge. In this way, "knowledge is more a novelty than functioning as an integrated whole" (Meyer, 2003b, p. 251); knowledge is sought for the sake of knowledge.

In order for this knowledge hierarchy to exist, it is essential that knowledge "exist [as] pieces of matter that can be discovered, uncovered, manipulated and sorted. The knower, the known, and the process of knowing can be thought of separately" (Malcolm et. al., 2008, p. 617). Thus, in the Traditional Western worldview, knowledge is fragmented, abstracted from the specifics of the context, and then put into categories that allow the value of the knowledge pieces to be assessed and placed appropriately within the hierarchy. A consequence is that "knowledges are organized around the idea of disciplines and fields of knowledge" (Smith, 1999, p. 65) – the constituent levels of the hierarchy – and, "While disciplines are implicated in each other… they are also insulated from each other through the maintenance of what are known as disciplinary boundaries" (p. 67). Knowledge of value within the Traditional Western worldview is thus fragmented, sorted, compartmentalized, and isolated.

**Dichotomization.**

The Traditional Western worldview also defines the worth of different kinds of knowledge in terms of dichotomies. The dichotomization of humans and nature and knower from object (Shelbert, 2003) ultimately leads to the belief that "mind and body must be separate if knowledge [is] to ever be trusted" (Meyer, 2003b, p. 12). Since knowledge is the key to personal progression up the knowledge and knowers hierarchies, this separation becomes yet another dichotomy that is bound to the Traditional Western worldview. In fact, Shelbert (2003) explains how the Traditional Western worldview is grounded in "'a mind-set of dualisms which claims an all pervasive polarity of what is', the duality of positive and negative, of human and divine, of humans and nature, of right and wrong…" (p. 63). Nisbett (2003) adds to the list of embraced dichotomies that of "the external, objective world and the internal, subjective one" (Science and Mathematics in Greece and China section, para. 1). Thus, within the Traditional Western worldview, "dualism is everywhere… and it is always oppositional and hierarchical, never neutral" (Strega, 2005, p. 203). By viewing knowledge through a framework of dichotomies, certain knowledges naturally "rise to the top" while others "sink to the bottom" of the hierarchical system of the Traditional Western worldview.

From the valuing of separation of both mind and body, and the external and the internal, comes the dichotomizing of subjective and objective knowledge, with a valuing of objective knowledge as knowledge that comes from the mind of the knower about an external object. The Traditional Western worldview holds that "Objectivity… can only be achieved through the application of reason, and therefore can be applied only by those who are rational" (Strega, 2005, p. 202). Since the sought-after objective knowledge is isolated from knowledge that comes from the body or that is internal, connecting the knowledge and knower, it is important that data be collected "without any understanding of its context and without any personal connection or stake in the data" (Absolon, & Willet, 2005, p. 105) so that it results in valuable knowledge. In this way, not only is knowledge of value fragmented and categorized, it is also abstracted to a general universally applicable fact. Conversely, subjective knowledge, knowledge that relates knower to object, mind to body, and human to nature, is not valued within the Traditional Western worldview.

**Rationality and universality.**

Within the Traditional Western worldview, "'facts' speak for themselves" (Smith, 1999, p. 31). That is, the rational knowledge of the Traditional Western worldview is thought to be indisputable, and because of these facts and their associated rules, people with such knowledge "can control events because they know the rules that govern the behavior of the objects" (p. Nisbett, 2003, Introduction section, para. 1). The basic premise behind the valuing of facts is that "things don't change much, or if they are really changing, future change will continue in the same direction, at the same rate, as current change" (Nistbett, 2003, Stability of Change section, para. 3). With things not changing, or if they do change they do so in a known direction, the hierarchy of knowledge, of facts, can remain intact and universal.

The notion of universality in the Traditional Western worldview extends beyond the facts of objects, to the assumption that "there are fundamental characteristics and values which all human subjects and societies share" (Smith, 1999, p. 30). Consequently, the facts of the Traditional Western worldview are assumed to be universal to all people, places, and times. This is possible within the Traditional Western worldview because "Everyone has the same basic cognitive processes … same tools for perception, memory, causal analysis, categorization and inference" (Nesbitt, 2003, Introduction section, para. 3a). Combining the notions of universality and decontextualization of knowledge results in the Traditional Western worldview holding that "scientific knowledge is superior to local knowledge" (Van Eijck, & Roth, 2007, p. 930), as local knowledge pertains only to local contexts; whereas, scientific knowledge is meant to reflect universal ideas and conditions. Therefore, not just any facts are sought, but scientific facts, because science claims: "that's the way it is" (Little Bear, 2000, p. 82).

**Scientific method and knowledge.**

Inevitably, because of the features of the Traditional Western worldview that value mind (rational thought) over body, objective over subjective, and external over internal, the high value placed upon scientific knowledge should not be surprising, as the scientific method holds the same values. Within the Traditional Western worldview, "science is the 'best' kind of knowledge, superior to various forms of unreliable and unverifiable non-scientific knowledge" (Strega, 2005, p. 202). The first way that science is able to achieve its supremacy as a way of knowing is that: "observation is attempted in isolation and in an artificial environment" (Little Bear, 2000, p. 82). By isolating the object both from the knower and the environment in which it naturally occurs, all outside influences on the knowledge sought can be eliminated. Thus, scientific knowledge, emerging from the scientific method, is "free of bias" and as such it is "positioned as the only kind of knowledge that can be relied upon for tasks that require prediction and control" (Strega, 2005, p. 202), a desired quality within the Traditional Western worldview. Further, scientific knowledge, and the scientific method, are valued most within the Traditional Western worldview because of the incorporation of both observation and measurement; since, "observation by itself is not good enough … If something is not measureable, then it is not scientific" (Little Bear, 2000, p. 83). This emphasis on measurability is important because it ensures the right kind of knowledge is being sought: physical processes and objects can be measured while anything subjective cannot. Thus, by insisting on scientific knowledge to be based upon measurement, the dichotomy of subjective and objective knowledge within the Traditional Western worldview is preserved: "Science sees the notions of intuition and feelings as something that 'dirties' knowledge, objective reality, and pure reason" (Meyer, 2003b, p. 12).

Verification is also central and crucial to scientific knowledge and the scientific method. "Science becomes truth or 'verifiable knowledge' through the stringent application of various tests. These verification methods include observation, mathematical calculation, experiment, and replication" (Strega, 2005, p. 202). Replication of scientific studies ensures the truth of the knowledge determined. Ultimately the Traditional Western worldview holds, "The application of rigorous scientific methods that derive from mathematical logic ensures objectivity, neutrality, and the absence of bias" (Strega, 2005, p. 202). These properties of scientific knowledge result in a "collective knowledge that is coherent with any observation of this natural world and that, as far as we know, counts everywhere irrespective of local contexts" (Van Eijck, & Roth, 2007, p. 930). Thus, scientific knowledge is highly valued within the Traditional Western worldview because it comes from physical observation, measurements, and logical reasoning within a bias-free environment by a knowledge seeker who is distanced from the object to be known. Credibility is further gained for these truths based upon the ability of any other person to replicate the findings of the scientific study. In addition, because the knowledge is gained in a position of isolation, it is abstracted from any context and is thus universally applicable.

**Linearity, singularity, objectivity, and staticity = power and authority.**

The emphasis placed upon scientific knowledge creates the necessity for knowledge of value to be viewed, as Little Bear (2000) describes, as not only objective, but linear, static and singular as well. The foundation of linearity emerges from the hierarchies of knowledge and knower, with progression upward through the hierarchies viewed as desirable, leading to the pinnacle of all knowledge and humans. Further, “Socially, [linearity] manifests itself in terms of bigger, higher, newer, or faster being preferred over smaller, lower, older, or slower” (Little Bear, 2000,p. 82). Once again, linearity emphasizes the existence of dichotomies, with “even the either-or orientation of [the] logic” (Nisbett, 2003, Philosophy in Greece and China section, para. 6) resulting in a linear progression of actions, decisions, and knowledge. The assumed linearity of knowledge of value also results in specialists who can study and know about each different stage within the hierarchical linear trajectory of knowledge, the result of which is “a social structure consisting of specialists. … Specializations are ranked in terms of prestige” (Little Bear, 2000, p. 82). Within the Traditional Western worldview, then, the more specialized and abstracted the knowledge a knower has, the higher up in the hierarchy of humans the knower is, and the greater authority and power that person has over others and their world.

Through the combination of linearity, specialization, and dichotomy, singularity of knowledge emerges “in concepts such as one true god, one true answer, and one right way” (Little Bear, 2000, p. 82). This singularity of existence and truth results in the Traditional Western worldview to hold that “certainly [it is] only rational ideas, the only ideas, which can make sense of the world, or reality, of social life and of human being” (Smith, 1999, p. 56). In this way, the singularity of knowledge translates into being the power and authority of the knowledge valued within the Traditional Western worldview, “[conveying] a sense of innate superiority” (p. 56) to all those who hold such knowledge. It is the seeking of singularity within one’s knowledge that brings a person grounded within the Traditional Western worldview to the search for “the ultimate truth, the ultimate particle out of which all matter is made” (Little Bear, p. 82). Moreover, because this is *the* particle out of which all matter is made, what is also being sought must be static knowledge, knowledge without change, knowledge without need for revision. Linearity, singularity, and being static in nature are inherently crucial within the hierarchy of the knowledge valued by the Traditional Western worldview.

**Preservation of knowledge through writing.**

As the knowledge of value within the Traditional Western worldview is factual, singular, static and objective, with the hierarchical linearity of it playing an important role in the development and learning of knowledge, it is necessary to, likewise, record all such knowledge in a way that is also remains constant.  To this end, the Traditional Western worldview recognizes writing (both in words and symbols) as the means to preserve knowledge of value. Recording knowledge in written form promises to keep permanently the knowledge as abstracted – objective truths – available to all to access and apply, provided they access it rationally, to their own lives as well as to build new knowledge of value upon.  In fact, "Writing has been viewed as the mark of a superior civilization" (Smith, 1999, p. 29), and since the knowledge of value within the Traditional Western worldview is superior to all else (because it is necessarily truth obtained in the right way), writing, like the scientific method is as a way of knowing, is the perfect fit when the documenting of knowledge is considered within this worldview.

**Summary of the Traditional Western worldview.**

In summary, the Traditional Western worldview relies upon science (and the scientific method) to determine knowledge of value about objects that are external to the knower.  This knowledge, which is objective, static, singular, and linear in relation to other knowledge is preserved in written form where it is sorted and categorized into hierarchical levels.  In this worldview, knowledge is sought for the sake of knowledge, as it is through the creation and gaining of knowledge that is higher within the hierarchy that prestige, control, authority, and power are gained by the knower.  Through the processes of fragmentation and categorization, knowledge becomes abstract and therefore universal, applicable to all contexts in which the object of study might be found.

## An Indigenous Worldview

Unlike the Traditional Western worldview, identifying an Indigenous worldview might, at the outset, seem like an impossible task, as there are many different Indigenous cultures from all around the world. In fact, it is for this very reason that I have chosen to name the worldview I am about to describe *an* Indigenous worldview rather than *the* Indigenous worldview – it is one of many possibilities and realities. Many of the different Indigenous cultural worldviews, however, are based upon a very similar set of understandings related to what knowledges and ways of knowing are valued. As a consequence, the worldview I am about to describe is general enough for many, perhaps most if not all, Indigenous worldviews to find a home within it. In many cases, the only variations between an Indigenous worldview and other worldviews related to Indigenous cultures is in how the worldview is enacted, such as through ceremony and protocols. As evidence of this similarity, in this description of an Indigenous worldview I will be referencing African, Alaska Native, Asian (in particular Chinese), Blood Blackfoot, Maori, Native Hawaiian, Plains Cree, Salteaux, and Woodlands Cree worldviews. Again, it must be emphasized that *an Indigenous worldview* is not the worldview as lived by any particular Indigenous people, but a conglomerate of commonalities found between multiple Indigenous worldviews. Furthermore, this worldview can neither be assumed to represent the worldview of any Indigenous person, nor can it be assumed that this worldview cannot be held by a non-Indigenous person.

Sternberg, Barrett, Blood, Glanfield, Lunney Borden, McDonnell, Nicol, and Weston (2010) identify "seven principles of reverence, respect, reciprocity, responsibility, synergy, interrelatedness and wholism" (p. 8) as foundational to Indigenous knowledge and ways of knowing. These principles result in the convictions: "(a) place educates, (b) beauty develops our thinking, and (c) time is not simply linear" (Meyer, 2003a, p. 251). These same ideas are echoed by Kovach (2009) when she writes "Descriptive words associated with Indigenous epistemologies include interactional, interrelational, broad-based, whole, inclusive, animate, cyclical, fluid and spiritual" (p. 56) ways of thinking. Like for many Indigenous Africans,

> commitments to the collective-self … relationships, harmony, and context see knowledge quite differently, more as a process than discovery, more as a relationship than entity, and more as verb than noun. At the same time, those commitments do not require us to reject the idea of knowledge as entity; rather we need to take account of context and purpose. (Malcolm, Sutherland, & Keane, 2008, p. 617)

Within these four accounts by four different sets of authors and researchers, the parallels and continuities between the worldviews that they are speaking of are evident. In fact, even in a Chinese worldview "concerns about harmony, holism, and the mutual influence of everything on almost everything else" (Nisbett, 2003, Philosophy in Greece and China section, para. 9) are present. In each of the distinct worldviews just referenced, there are many different common factors that influence what kinds of knowledge and ways of knowing are of value, but central to them all is the notion of relationship.

Within an Indigenous worldview, establishing, strengthening, and maintaining relationships with all of creation (including people, the earth, nature, the spirit world, and the cosmos) is foundational to the creating, gaining, and sharing of knowledge – knowledge "[results] from interactions with the group and with all of creation" (Little Bear, 2000, p. 79). Significantly, human beings are considered part of the animal kingdom, "humans are not regarded as more important than nature" (Snively, & Coriglia, 2001, p. 12), and thus they are not designated any special position, authority, or power in relation to knowledge or use of it. Moreover, an Indigenous worldview sees "The natural world as one large system that is inseparable from human experience" (Roy, & Morgan, 2008, p. 237); therefore, how humans relate to the natural world has consequences for the human experience. Within the notion of relationship, therefore, an Indigenous worldview holds that humans are an integral part, but no more significant than any other, of the relationships that serve to create and share knowledge.

**Relationship, context, and the whole.**

Relationship holds this central positioning within an Indigenous worldview because this worldview considers the world and experiences within it as wholes. That is, an Indigenous worldview considers the world from the perspective of complex contexts and events, finding "the world [is] simply too complex and interactive for categories and rules to be helpful" (Nisbett, 2003, Science and Mathematics in Greece and China section, para. 8). This holistic view of experiences and ultimately knowledge means that within an Indigenous worldview, "coming to know the natural world… is not a separate discipline in isolation from everyday living. Learning takes place within the messiness and complexity of life" (Michell, 2005, p. 38). In coming to understand an Indigenous worldview, then, one must explore what is considered important when relating with this messiness and complexity.

**Relationship and place.**

Within an Indigenous worldview, place is also always of paramount significance. It is important that relationships, and the resulting knowledge that emerges and is preserved through them, occur and be associated in particular contexts and places, and not in isolation. Michell (2005) says of the importance of place "our lives, stories, experiences, challenges, births and deaths are written all over this landscape" (p. 35). There is a fundamental recognition that place influences who people are and what they do, and objects are understood within the contexts that they are created, found, or used. Simply put, "Place matters" (Meyer, 2003b, p. 143).

Consequently, since place matters, and knowledge is in relationship to place, then place is a source of knowledge. As the source of knowledge, it is therefore important within an Indigenous worldview that knowledge "never be taught in a decontextualized format; it must be tied to the land and the community" (Malcolm et. al., 2008, p. 619), it must be tied to place.

Many authors and researchers (for example, see Kovach, 2009; Little Bear, 2000; Meyer, 2003b; Smith, 1999) connect an Indigenous worldview's emphasis upon place to the importance of knowledge of the land to survival for Indigenous people. As Meyer (2003b) explains, "The quality of our survival was tied to the intimate knowledge we had of nature's moods, planting secrets, weather patterns, history and seasonal temperaments. The environment: plants, wind, stones, rains – they were the stuff of poetry, wisdom, healing, food, inspiration" (p. 98), making place the centerpiece for all knowledge. Within an Indigenous worldview, place is central to relationships because the establishment, strengthening, and maintenance of relationships all happen within place.

**Relationship, ways of knowing, and diversity.**

The acceptance of the messiness and complexity of place, and the knowledge related to place, also connects to the understanding within an Indigenous worldview that "there are both physical and spiritual laws that govern the universe. What is done in one realm is mirrored in the other. The sacred and the secular are not separate" (Michell, 2005, p. 37). Meyer (2003b) emphasizes the connection amongst relationships, knowledge, and spirituality when she writes: "Knowledge/spirituality were interwoven in almost every description of how Hawaiians viewed intellect, skill acquisition, wisdom, learning, knowledge and understanding" (p. 93). Sacred knowledge has this important role in an Indigenous knowledge because "In Aboriginal philosophy, existence consists of energy. All things are animate, imbued with spirit, and in constant motion. In this reality of energy and spirit, interrelationships between all entities are of paramount importance, and space is a more important referent than time" (Little Bear, 2000, p. 77). Spirituality is the result of relationships with the land (Michell, 2005) and "ancestors, both alive and dead" (Meyer, 2003b, p. 93). Spirituality becomes part of relationships through "the quest for visions" (Ermine, 1995, p. 109), intuition (Kovach, 2009), "environmental signs" (Meyer, 2003b, p. 104), "prayer" (Ermine, 1995, p. 109), "dreams" (Meyer, 2003b, p. 166), and other cultural ceremonies and protocols. An Indigenous worldview does not insist on each of these ways of being within a spiritual or spiritually impacted relationship; moreover, an Indigenous worldview is broad enough to accept any way in which the sacred might be brought into relationships. In an interview with Margaret Kovach, Graham Smith commented "that the Maori do not have the same traditional beliefs around dreams, but he would not dismiss this as a valid knowledge source" (Kovach, 2009, p. 58). Regardless of the origins of the sacred for Indigenous peoples, spiritual relationships arise within "the inner space in the individual [giving] rise to a subjective world-view out onto the external world" (Ermine, 1995, p. 108). Thus, spirituality, as a source of knowledge within an Indigenous worldview also connects relationship to the valuing of diversity in knowledge and ways of knowing.

From the perspective of an Indigenous worldview, relationships, and as a result the knowledge they embody, can take on any form, for example, physical, emotional, spiritual, and intellectual. Since knowledge is created and shared through relationships that are not just intellectual in nature (but also emotional, physical, and spiritual), the criteria that knowledge of value come from objective sources and processes is not found within an Indigenous worldview (Ermine, 1995; Kovach, 2009; Little Bear, 2000; Meyer, 2003b; Smith, 1999). Instead, subjective knowledge is often viewed as just as valuable as objective knowledge – the value depending upon the context within the knowledge is gained, shared, or to be applied. Personal experience and intuition (based upon the past, present, and future) are also considered valid sources of knowledge within an Indigenous worldview (Ermine, 1995; Kovach, 2009; Little Bear, 2000; Meyer, 2003b; Smith, 1999).

An Indigenous worldview recognizes that, "'Even within our species, our minds, memories, and personal experiences make us diverse from each other (McGaa, 2004, p. 14)'" (cited in Michell, 2005, p. 35). Further, because of viewing all of aspects of the natural world as interdependent, an Indigenous worldview also respects and values cultural diversity. Within this worldview, it is recognized that "a certain practice that works for one person may not necessarily work for another person. Openness and flexibility is encouraged" (Mitchell, 2005, p. 40). For Hawaiians', "epistemology is the *study of difference*. Because formulating ideas in Hawaiian epistemology needs contrasts from which to emerge" (Meyer, 2003b, p. 76), diversity in knowledge and ways of knowing are not only accepted, but sought: "I tell this story to remind myself to not simply 'tolerate diversity.' We should, instead, be fully engaged, changed and humbled by it" (p. 4). An Indigenous worldview seeks out and celebrates a "Multiplicity of sources comprising Indigenous ways of knowing" (Kovach, 2009, p. 78) including the intuitive, subjective, emotional, physical, spiritual and intellectual ways and kinds of knowledge mentioned previously. Therefore, "Each being ought to have the strength to be tolerant of the beauty of cognitive diversity" (Little Bear, 2000, p. 80) as it is foundational to an Indigenous worldview.

Within an Indigenous worldview, the five senses, touch, sight, hearing, smell and taste, as part of physical ways of knowing, are also valued sources of knowledge. This "idea that knowledge comes from our five sense is shaped by a distinct relationship we have had with the world as cultural people" (Meyer, 2003b, p. 110). Sensory knowledge is not only gained from one's personal physical experiences. It is also mediated "by a whole host of historical and metaphorical images that continue to explain, educate and inspire" (p. 107) through stories of place and context. Thus, sensory knowledge belongs both to the sacred and the secular realm, to inner and outer space. This notion is confirmed by Aboslon and Willett (2005) when they write: "Memory is more than a mental process of recalling facts, experiences, and information. Human beings also have a capacity for sensory, physical, spiritual, and emotional memory" (p. 116). Thus, the five senses play an important role in the creating and sharing of relationships, and therefore knowledge, within an Indigenous worldview.

**Connectedness: mind and body, objective and subjective.**

The valuing of relationships based upon these different ways of knowing, and the knowledge that is bound within those relationships and their places, inherently implies that the mind and the body are not separate entities: "The mind is body and the body is mind. … Thinking and feeling are not separate" (Meyer, 2003b, p. 124). In fact, "For Hawaiians, separation of [mind and body is] an illusion: the stomach region [is] indeed the seat of emotion as well as the seat of intellect" (p. 123). As a consequence, within an Indigenous worldview, "exploring existence subjectively" (Ermine, 1995, p. 104) is valued as well as doing so objectively. It is place, and the relationships connected to place, that determine whether subjective, objective, or both kinds of knowledge are of value.

**Relationship, flux, cycles, and a holistic view.**

Part of the messiness and complexity of relationships within an Indigenous worldview is the recognition that "The world is constantly changing" (Nisbett, 2003, Philosophy in Greece and China section, para. 10). Consequently, "Indigenous knowledge and culture is dynamic – ever flowing, adaptable, and fluid. … opinions, thoughts, ideas, and theories are in a constant flux" (Aboslon, & Willett, 2005, p. 111). The recognition of the flux and the valuing of wholeness ultimately confirm and support each other: "The value of wholeness speaks to the totality of creation, the group as opposed to the individual, the forest as opposed to the individual trees… the constant flux rather than on individual patterns" (Little Bear, 2000, p. 79); wholeness highlights flux, and flux encourages one to focus on the whole.

As well, the focus on flux discourages (but, in accepting diversity, does not deny) abstraction and compartmentalization – keeping emerging and shared knowledges relevant and meaningful through place. To this end, Michell (2005) writes: "Cree people also do not sub-divide and fragment the natural world into small units such as biology, chemistry and physics; rather, the Woodlands Cree regard all life as being mutually independent" (p. 37). Further, the notion of flux not only leads to a holistic perspective, but also a "cyclical view of the world" (Little Bear, 2000, p. 78), where repetitive patterns within the whole help to create and clarify knowledge. As a consequence of a focus on cycles and patterns, "For Indigenous people there is a recognition that many unseen forces are at play in the elements of the universe and that very little is naturally linear, or occurs in a two-dimensional grid or three-dimensional cubic form" (Barnhart, & Kawagley, 2005, p. 12). Ultimately, these understandings of flux, cycles, patterns and non-linearity, also lead to values regarding other aspects of knowledge and ways of knowing as well.

**Relationship and process.**

One consequence of an Indigenous worldview's recognition and valuing of the whole, flux, cyclical patterns, and non-linearity is that it also places greater emphasis on process than product in relation to knowledge creating and sharing (Absolon, & Willett, 2005; Meyer, 2003b). Kovach (2009) explains that it is "a worldview that focuses as much, if not more, attention on process than on product or outcome" (p. 66). It is important to note however, that products (outcomes) are not considered unimportant unless the process is not correct or done properly, such as within relationships and respecting diversity.

**Relationship and time.**

Another impact of the seeking and valuing of cyclical patterns within an Indigenous worldview is that "Time is not simply linear" (Meyer, 2003b, p. 63); rather, it "is dynamic but without motion. Time is part of the constant flux but goes nowhere. Time just is" (Little Bear, 2000, p. 78). I am sure that for many people, including myself, this view of time is perplexing to say the least; however, I have come to understand through these ideas that time does not need to dominate knowledge nor the relationships (to others and to place) that ground knowledge because of the cyclical patterns of the flux. Therefore time, specific linear time, need not have any impact upon the knowledge emerging from relationships tied to place within an Indigenous worldview.

**Relationship and dichotomy.**

The recognition of flux within the whole also brings out specific types of relationship in an Indigenous worldview amongst the knower, knowledge, and dichotomies: "The world is constantly changing and is full of contradictions. To understand and appreciate one state of affairs requires the existence of its opposite; what seems to be true now may be the opposite of what it seems to be" (Nisbett, 2003, p. 12 of 217). In other words, like the separation of mind and body, within an Indigenous worldview, dichotomies are an illusion. Even the separation of mind and body is seen as an illusionary dichotomy. In an Indigenous worldview, "all dualities merge and knowledge becomes less a *thing* than an event" (Meyer, 2003b, p.253); in fact, within an Indigenous worldview, it is assumed "that we can hold polar truths as part of one truth" (p. 66). Thus, seemingly dichotomous notions, such as good and evil, become part of the whole, contributing to the flux and cycles of knowledge and relationships, they are "vital sequences into the all encompassing Self that unfolds through time and space" (p. 66), being recognized and valued as part of the diversity of the complexity and messiness of knowledge, relationships, and life. Biases, as dichotomous ideas to something else, then are also not actively sought out and eliminated in an Indigenous worldview; rather, they are recognized as "an energy to be directed toward good" (Meyer, 1998, p. 26).

**Relationship, reciprocity, and the greater good.**

An Indigenous worldview also seeks knowledge that is useful making "everything learned something of value" (Meyer, 2003b, p.113). Both Nisbett (2003) and Meyer (1998) explain that knowledge is sought to fulfill a need and not for the sake of knowledge itself; however, "This does not suggest that utility did not have some aspect of aesthetic or psychic relevance" (Meyer, 2003b, p. 113). Utility and the aesthetic and psychic are not positioned as a dichotomy; rather, they were viewed as a likely pairing within all knowledge of value.

Since knowledge of value is seen as one that fulfills a need within a particular place and relationships, reciprocity is a natural part of the seeking and gaining of knowledge within an Indigenous worldview: "The Cree ethic of reciprocity teaches that what you take you must share and give back" (Michell, 2005, p. 37); thereby, ensuring the continuation of good relationships. This also implies that a "community's local Indigenous knowledge … [cannot] just be taken; it is a body of knowledge that must be earned" (Malcolm et. al., 2008, p. 614). Moreover,

> As gifts of truth and knowledge are shared, we take what we need out of respect in order to become whole and complete. We decide what works in our own lives, what works for a specific situation, and then determine what needs to be left behind. (Michell, 2005, p. 36)

What is taken from the knowledge is what is needed for the current situation – the place – and as such new knowledge is formed in relation to this place. Because place determines what knowledge is shared and what knowledge is taken, the way in which the knowledge is shared is of great importance, as it needs to have the ability to adjust according to the variability of the world.

**Relationship and orality.**

Within an Indigenous worldview, the way in which knowledge is held to be best shared is through oral traditions. By housing knowledge within oral stories, it is possible to "tell our stories one way today, then revise and retell them tomorrow" (Absolon, & Willett, 2005, p. 112). Whereas writing freezes what is known and how it is shared, oral approaches to knowledge allow the sharer of that knowledge to adjust it to meet the needs of the person or people receiving that knowledge. In this way, knowledge can be adjusted to better align with the particular place and relationships of the knower. This aspect of an Indigenous worldview is also connected to the acceptance and seeking of diversity in knowledge, as it recognizes that knowledge, in its entirety, is not transferable because of the fluidity of place and relationships. Meyer (2003b) adds: "When stories are shared, they are filtered through the listener's own historical lens, sensuous training, gender and political context" (p. 141), yet again connecting oral transmission of knowledge to the valuing and acceptance of diversity. It is not expected that all people will take the same knowledge from a story; rather, it is expected that all knowledge will be reformed by both the speaker and the listener to meet the needs of the listener and knower.

**Summary of an Indigenous worldview.**

Little Bear (2000) speaks an Indigenous worldview as being based upon wholeness: "like a flower with four petals. When it opens one discovers sharing, honesty, and kindness. Together these four petals create balance, harmony, and beauty" (p. 79). These desired traits resonate with and emerge from within an Indigenous worldview through its valuing of knowledge that is based in positive and meaningful relationships with all of creation grounded within place. Such knowledge includes the subjective and the objective, the sacred and the spiritual. This knowledge is flexible and diverse, responding to changes in place and relationships as well as being informed through many ways of knowing (spiritual, emotional, physical, intellectual, intuitive, experiential, cultural…). An Indigenous worldview does not separate humans from the rest of nature, nor does it separate the mind from the body; instead, dichotomies are embraced as sources of diversity and possibility. All knowledge of value within an Indigenous worldview is sought in response to need, the need to survive and the need to live in balance, harmony, and beauty. This knowledge is remembered and shared through oral stories and traditions so that it might remain flexible, able to change according to the cyclical flux of the world.

## The Two Worldviews: Implications and Concerns

As is by this point undoubtedly evident, these two worldviews do indeed provide stark contrast (Smith, 1999) to one another. Whereas within the Traditional Western worldview each foundational belief builds upon the linearly and hierarchically previous ones, the foundational beliefs of an Indigenous worldview are directly tied to all others through the cohesion of relationships. The result is two worldviews that are built upon very different premises: interconnectedness and isolation – one (the Traditional Western Worldview) is looking for linear trajectories of knowledge, while the other (an Indigenous Worldview) is seeking knowledge within cycles of patterns. The implications of these differences as a result of these two worldviews interactions are beyond the scope of this dissertation; however, there are a few points in this regard that I would be remiss in not presenting here.

As Nisbett (2003) points out,

> there is great potential for conflict when people from cultures that have different orientations must deal with one another. This is particularly true when people who value universal rules deal with people who think each particular situation should be examined on its merits and that different rules might be appropriate for different people. (Independence vs Interdependence section, para. 24)

The conclusion commonly reached (or assumed) is that one group must give in to the "wisdom" of the other. Therein lies a further complexity to this situation, for one group, in allowing for diversity of thinking, is forced to give way to the other group which allows only for one conclusion; thus, an Indigenous worldview would ultimately (be expected, at least) to acquiesce to the Traditional Western worldview. This is a consequence of the Traditional Western worldview's conviction that it's knowledge is absolute truth, and must therefore be applicable to every situation. Strega (2005) notes: "The hegemony of the world view [(the Traditional Western worldview)] is more than one way to view the world; it is successfully positioned as the most legitimate way to view the world" (p. 201). It is the lack of emphasis on singularity of knowledge within an Indigenous worldview that has allowed the Traditional Western worldview to claim that their singularity is superior.

Snively and Corsiglia (2001) explain that this same authority housed within the Traditional Western worldview allows its knowledge to then be a "vigorous 'gatekeeper' that has certainly succeeded in screening out metaphysical, pseudo-science" (p. 9) in order to protect the hierarchy of truth it has produced. In this way, the Traditional Western worldview is "indifferent to the processes of Indigenous knowledge" (Battiste, 2002, p. 17), holding "the subjective as an unwanted stepchild to objective data" (Meyer, 2000a, p. 252). This central assumption of the Traditional Western worldview therefore is able to create an illusion of superiority over the knowledge of an Indigenous worldview, and thereby "disrupts the balance" (Michelle, 2005, p. 36) for the holders of the Indigenous worldview and between the two worldviews.

The differences between the Traditional Western worldview and an Indigenous worldview ensured the door to colonization (and it's impacts) was wide open: "Indigenous peoples were classified alongside the flora and fauna; hierarchical typologies of humanity and systems of representation were fuelled by new discoveries; and cultural maps were charted and territories claimed and contested by the major European powers" (Smith, 1999, p. 1). Because of the kinds of knowledge and ways of knowing valued within the Traditional Western worldview, colonizers were able to relegate Indigenous peoples (from around the world) to subhuman members of the hierarchy of knowers and of humans (Smith, 1999). It also allowed the Indigenous subjective (or subjective and objective) knowledges of place and relationships to be converted into compartmentalized and abstracted truths to be stored within the Traditional Western hierarchies. Ultimately, the power and authority assumed by the Traditional Western worldview allowed colonization to "[suppress] the diversity of human worldviews" (Little Bear, 2000, p. 77), "confining Indigenous people to alienation in perpetuity" (Henderson, 2000, p. 69). Colonization, and all of the consequences for both colonizers and the colonized are directly (along a linear trajectory) related to the Traditional Western worldview and its all but eradication of an Indigenous worldview (Henderson, 2000; Little Bear, 2000; Meyer, 2000b; Smith, 1999).

When considered from the perspective of specific aspects of life, such as law, education, medicine, and so on (note: these are all Traditional Western categorizations of aspects of life), the results are the same – when the two worldviews meet, the Traditional Western worldview assumes power and authority while an Indigenous worldview is virtually dismissed as pointless in the process. However, I believe that the antagonistic relationship between these two worldviews is a possible source of both frustration and of possibility.

**Concluding Thoughts on the Worldviews**

Becoming aware of these two worldviews has made me curious about how each would view my story about mathematics and me. Nisbett (2003) claims: "The Westerner sees an abstract statue where the Asian sees a piece of marble; the Westerner sees a wall where the Asian sees concrete" (Holism vs section, para. 6). Looking through the two lenses of an Indigenous worldview and the Traditional Western worldview at my story, what can be seen through each? Further, what does mathematics and the teaching and learning of mathematics look like through each lens? What impacts would grounding mathematics and the teaching and learning of mathematics in the Traditional Western worldview or an Indigenous worldview have upon Indigenous (and other) students? How might mathematics itself be changed, improved, or fortified through worldviews?  Should we, in education and in society, be changing the lens through which we view mathematics and the teaching and learning of mathematics, or is there even a way, despite that the two "forms of knowledge are incommensurable with each other" (Van Eijck, & Roth, 2007, p. 935), for the two worldviews to exist in harmony, or at least in a state of cooperativeness and non-interference?  These reflective questions, which have emerged from my coming to better understand these two worldviews, have resulted in the formulation of my research question:

*What ways of knowing and kinds of knowledge are, and possibly could be, valued within mathematics and the teaching and learning of mathematics?*

## Methodology and Methods

With my story told and the theoretical worldview framework discussed, I can now move into a discussion of the methodology and methods that I will be using within my research. The forward positioning of the framework and of my story (even the inclusion of my story at all), may seem unconventional; however, there is purpose behind what I have done. In order to answer my research question, there are three different methodological approaches (auto/ethnography, Gadamerian hermeneutics, and grounded theory) that I will be calling upon, and it is within my discussion of these methodologies that one will find justification and purpose for both my story and the theoretical framework.

If I were to follow the naming tradition within qualitative research, what I propose as my research methodology is actually a "bricolage" (see Denzin & Lincoln, 2011) of auto/ethnography, Gadamerian hermeneutics, and grounded theory; however, the word "bricolage" does not fit how I envision the formulation of my research methodology. Bricolage, as defined by Merriam-Webster (2015) is "construction (as of a sculpture or a structure of ideas) achieved by using whatever comes to hand; *also* something constructed in this way." WiseGEEK (2016) gives the following extended definition: "Bricolage is a word which is used to mean an assemblage of objects, along with the trial and error process of putting such objects together." Further, The Association for Qualitative Research (2015) calls methodological bricolage "a pragmatic and eclectic approach to qualitative research". What I am proposing for the methodological framing of my research is *not* constructed of whatever might happen to be available, it is *not* the result of a trial and error process, and its contributing methodologies are *not* eclectic, as they braid together with meaning and purpose. I prefer, instead, to refer to my overarching methodology as a collage, "a combination of a collection of various things to create a new whole" (Merriam Webster, 2015). I also choose not to name this collage, as it is contextually bound, and even if the same three methodologies were to be encompassed in another methodological collage, it would inherently be different because of the difference in the context of use and the difference of user (even if that user is myself). For me, the methodology I propose is purposeful and connected both internally amongst its component parts and externally with my research questions and data. In so saying, I now will introduce you to my collage, detailing the pertinent features of each of the three methodologies in play and then how they relate to one and another within the context of my research, starting with the methodology of auto/ethnography.

### Auto/ethnography

Before entering into a discussion of why I choose to use "auto/" over "auto" when writing of auto/ethnography, it is probably best that at least a general definition of what I mean by a methodology of auto/ethnography be given. Ellis, Adams, and Bochner (2011) define auto/ethnography as "an approach to research and writing that seeks to describe and systematically analyze (graphy) personal experience (auto) in order to understand cultural experience (ethno)" (History of Autoethnography section, para. 1). As a part my research I will be analyzing, through the lenses of the Traditional Western worldview and an Indigenous worldview, my story in order to try to better understand both the cultures of and experiences within mathematics and the teaching and learning of mathematics. In particular, I will be seeking to identify what kinds of knowledge and ways of knowing are being valued within mathematics and the teaching and learning of mathematics, and to explore both the impacts and possibilities as a result of that analysis. With this broad brushstroke understanding of auto/ethnography, I will now return to the questions of why "auto/".

#### Choices in naming.

Like W. -M. Roth (2005a), I choose to denote the methodology most commonly named autoethnography as auto/ethnography with the intention of emphasizing "What an individual does is always a concrete realization of cultural-historical possibilities" (p. 4). That is to say, in auto/ethnography, cultural-historical possibilities contribute to the research process as much as the story of one's self, because that story is always part of the cultural-historical possibilities: "The specifically human form of existence is possible only because of society" (p. 3). More directly stated, auto/ethnography is a "concretely realized form" (p. 4) of ethnography in general, with the concreteness being in the positioning of the ethnography within the context of one's self. Thus, in auto/ethnography, the focus is not solely on the auto, but also on the ethnography and biography and their cultural-historical possibilities. As a researcher employing auto/ethnography, my writing and analysis will naturally flow between and bring together the auto, the ethnographic, and the biographic. For me, the introduction of self into ethnography is not done through a merger; rather, it occurs through a meaningful dance between self, ethnography, and biography.

**Methodological differences.**

It is also important to note that my research differs from the auto/ethnographic research most frequently described and documented in the literature (for example, see Roth, 2005a) in two ways. First, I am not considering one culture and a set of related experiences, rather two: mathematics *and* the teaching and learning of mathematics. Some may argue that they are actually part of the same culture, but there has been enough discussion, especially in the last ten years or more (for example, see Schoenfeld, 2004), over whether the mathematics as perceived and done by mathematicians and the mathematics that is taught and learned in school are mutually compatible, let alone similar. In order to better understand this divide, if it exists, they must therefore be, at least initially, considered as two different cultures. This deviation from the "norms" of auto/ethnography should not be viewed as problematic; however, as the goal of my research remains the same as that found within the literature: "to study a culture's relational practices, common values and beliefs" (Ellis, et. al., 2011, Doing Autoethnography: The Process section, para. 3) emerging from the experiences shared within the researcher's own auto/biography. I am merely aiming to meet this goal twice, and then, hopefully, find a way to connect the results.

The second way that my research differs from other auto/ethnography research is in the way that I am not only analyzing my story about mathematics and the teaching and learning of mathematics from my own perspective, but I am also analyzing them by assuming the role of two "others" – one grounded within the Traditional Western Worldview and one grounded within an Indigenous worldview. This is important to note, because as Roth (2005a) warns: "the study of personal experience requires a radical suspension of judgment and submission to systematic method of dealing with one's own prejudices and prejudgments – lest … auto/ethnography are to lead to ideology, delusion, and conceptual blindness" (p. 9). As before, however, I do not see this variance as a cause for concern. Instead of focusing only on my own prejudices and prejudgments, I will also be revealing possible prejudices and prejudgments of two Others. In the analysis of my personal story, what appears as prejudices and prejudgments will depend upon whether I am looking through my personal lens, or one of the two worldview lenses. Each of these lenses will reveal prejudices in relation to the kinds of knowledge and ways of knowing that are to be valued, with my own prejudices very likely moving somewhat fluidly between those of the two worldviews. Moreover, the reader will respond differently to these differing values and associated prejudices and prejudgments based upon their own worldview. Overall, the revealing and interplay of all of these prejudices and prejudgments will broaden the reader's understanding of these two mathematically situated cultures, and in so doing, will continue and, more importantly, expand the meaningfulness and responsiveness of the resulting dialogue.

**Intentionality.**

There is one particular type of prejudice which Rodrigues (2005) argues definitely must be addressed by the auto/ethnographer (and researcher), namely intentionality: "*Intentionality* is defined here in terms of the consciously driven ideological, political, pedagogical, and theoretical motives behind the desire to tell a chosen story of self" (p. 121). In fact, Rodrigues contends: "the author's intentionality for sharing a chosen story of self should be as important as the telling itself. … This *intentionality* should be made visible to readers in order to reveal often taken-for-granted political and ideological agendas" (p. 122). It is through such revelations that "more spaces for critical engagement with the author's lived text are possible" (p. 123), in particular the space to critique that very intentionality. My story, at least indirectly, exposes my intentionality in pursuing this research: to understand, and expectantly find ways that may serve to rectify, the social injustices that the "outsiders" to mathematics and the teaching and learning of mathematics have to endure. Such social injustices include the social stigma of "failing" at mathematics, of not finding ways to relate ones life to mathematics, and the facing of the frequently insurmountable barriers that mathematics has created within their lives. Thus, my intentionality is focused upon understanding and confronting social injustices related to mathematics and the teaching and learning of mathematics.

**"I" as researcher and researched.**

Following from my intentionality, a significant feature (and point of contention within the world of research) of auto/ethnography is that it fundamentally "challenges canonical ways of doing research and representing others and treats research as a political, socially-just and socially-conscious act" (Ellis, et. al., 2011, Abstract section, para. 1). In particular, "The idea of an independence of the observer (and therefore his/her knowledge) and the world observed has been seriously questioned both in the natural and the social sciences" (Roth, 2005a, p.7). Auto/ethnography does this by recognizing and highlighting that the researcher is central to both the data and its analysis, it includes "I" as a central figure, overtly active within all aspects of the research. Within research in the natural sciences, the focus of research is typically on that which is outside of the researcher; the knower and what is coming to be known are methodologically, methodically, and purposefully kept separate from each other. When reading such research, the identity of the researcher is only known through the association of their name to the work, and not their presence within the analysis and documentation. Auto/ethnography, on the other hand, fully embraces the researcher as part of what is being researched; in fact, "in auto/ethnography, researchers constitute their own object of research so that the knowing subject and the research object become one" (Breuer, 2005, p. 109). Thus, "I" is prominently featured throughout such research, resulting in the exploration of the conditions and thoughts of the "insider" providing a more informed understanding of possibilities.

The prominence of "I" as a feature of this methodology speaks loudly to me. I have always felt that one could never completely remove their biases from their research activities. Who one is, and what they believe, necessarily influences how they interpret and understand things, and thus, I prefer approaches to research that choose to lay bare the researcher. In so doing, the reader might ascertain what biases and prejudices are at play in the research processes and engage in informed discussions and make informed decisions. Thus, as auto/ethnography researchers "assert the presence of the 'I,' the knower alongside the known in a dialectical knower/known relation, the knower and the known presupposing one another " (Roth, 2005a, p. 13), so too do I enter into this same kind of dialogue between knower and known. As noted before, however, "I" am not in my research as a singular I, but a multiplicity of Is – myself, as well as Others, who I have chosen because they and their knowledge are grounded within either an Indigenous Worldview or the Traditional Western Worldview.

As a warning to researchers considering the use of auto/ethnography as their methodology, Reed-Danahay (2009) states:

> Autoethnography is not the kind of autobiography in which the author as hero or heroine is neither constrained nor assisted in life by economic, social, or cultural position; autoethnography is also not a form of writing ethnography that erases the anthropologist and his or her encounters (Reed-Danahay, 2009, p 43)

from the research. Consequently, in my presentation of my auto/biography, my story, I have reported all that I can recall about my interactions with mathematics and the teaching and learning of mathematics, regardless of whether, from my current standpoint, I am proud or ashamed, pleased or disgusted, by my actions and thoughts. The "I" in the story is presented as I recall it, without embellishment or edits, it is my authentic self as I know it at this time. Further, in the analysis of the story, the reader will note that I will not attempt to distance myself from the "I" of the story, for that I is part of me and part of my history, and in choosing to engage in auto/ethnography, I am explicitly choosing to keep myself always within my research and the analysis therein.

**Researcher and reader as insiders.**

Adams, Jones, and Ellis (2014) further explain that in auto/ethnographic research,

> complex, insider accounts of sense-making and [illumination of] how/ why particular experiences are challenging or important and/ or transformative" (p. 27) are of central focus. By carrying out analysis of his or her auto/biographies, the researcher not only seeks to reveal what can be learned from his or her story, but also to demonstrate, through their own work, how others might also "make sense of similar experiences. (p. 27)

Through the processes of auto/ethnography, the researcher hopes to engage readers in an exploration of their own experiences and stories, and to respond, whether within their own personal lives or directly with the researcher, to what has been presented. "Autoethnographers invite participants and readers/ audiences to engage in the unfolding story of identities, experiences, and worlds, to creatively work through— together— what these experiences show, tell, and can mean" (Adams, et.al., 2014, p. 34) through both intrapersonal and interpersonal dialogue. For me, dialogue is one of the most important consequences that can result from any research because it leads to greater understanding and at times, agreement. Consequently, auto/ethnography speaks to me in a very personal way.

**Intersubjectivity.**

Another important point regarding engagement in auto/ethnographic researcher is that

> Researchers do not exist in isolation. We live connected to social networks that include friends and relatives, partners and children, co-workers and students, and we work in universities and research facilities. Consequently, when we conduct and write research, we implicate others in our work. (Ellis, et. al., 2011, Relational Ethics section, para. 1)

This is of course true within my current research. My story, in itself, explicitly invokes the spirit of others in my life, for example my grade one teacher (who taught me about the right way in mathematics), my father (who taught me a different right way to add columns of numbers), my vector calculus professor (who made me tea and showed me how little mathematics I understood), and my fellow mathematics major (who disastrously "hit and then saw the wall" of not understanding mathematics when it literally was too late), to name just a few. It is because of all of these people that I have this story to tell and it is because of these people that I came to do this research. Roth (2005a) argues these ways in which others, both directly and indirectly, are involved and implicated in auto/ethnographic research with us "are legitimate ways of establishing intersubjectivity that escapes the false dichotomy opposing objectivism and subjectivism" (p. 6), bringing both interpersonal and intrapersonal agreement to new understandings.

Auto/ethnography research seeks to find understanding from the perspectives of both insiders and outsiders to the culture, and potentially develop ways to bridge the divide between them. Within my research, the insiders in the cultures of mathematics and the teaching and learning of mathematics are those who were "successful" (in terms of school evaluations) and who have gone on to at least like, if not specialize, within at least one of the two named mathematics-centered cultures. The outsiders, on the other hand, are those who at some point stopped being (or painfully, may never have been) successful in mathematics and who feel disenfranchised from or diminished by their experiences with mathematics and the teaching and learning of mathematics. It is my hope that the analytic lenses of the two worldviews, focusing on what kinds of knowledge and ways of knowing are being valued (and not valued) in mathematics and the teaching and learning of mathematics, will create a deeper understanding of these two cultures, and more importantly, highlight possible inroads for those who currently sit outside them. Like Barton, and Darkside (2005), I "think about [auto/ethnography] as both the telling of one's story and the using of that story with others to understand and use difference productively" (p. 23).

**Reliability, validity, and generalizability.**

Some authors and researchers critique auto/ethnography in terms of its reliability, validity, and generalizing; however, Ellis, et.al. (2011) argue:

> Those engaging in auto/ethnography and auto/biography openly recognize the fallibility of memory over the passage of time. As such, the reliability, validity, and generalizability of one's auto/biography is construed in the same way as it is when the researcher is not part of what is being researched. (Ellis, et. al., 2011, Reliability, Generalizability, and Validity section, para. 1)

Thus, by overtly acknowledging the fallibility of this methodology within the research, the relative reliability, validity, and generalizability is determined by each individual who interacts with (or is indirectly influenced by) the research through their own particular lenses of prejudices and prejudgments. Auto/ethnography does not claim authority, universality, or absolute truth – only deeper understanding of complexities and awareness of possibilities.

**Reflexivity.**

Reflexivity is also an important aspect within auto/ethnography: "Reflexivity consists of turning back on our experiences, identities, and relationships in order to consider how they influence our present work. Reflexivity also asks us to explicitly acknowledge our research in relation to power" (Adams, et.al., 2014, p. 29). The first part of reflexivity, of looking at the personal experiences that have influenced my research I am conducting, is most obviously present in the construction of my research question. Without the examination of my past story, I would have no reason, other than possibly coincidence, to have created and engaged with this question. As Denzin (2013) states, "autoethnographic work must always be interventionist, seeking to give notice to those who may otherwise not be allowed to tell their story or who are denied a voice to speak" (p. 6), calling to attention and into question sites of power and authority within the culture being studied. I believe the most significant way that power is considered and engaged within my research is through my desire to understand what it is about mathematics and the teaching and learning of mathematics (specifically, what is or is not being valued within these two cultures) that is denying some people access to them, and in doing so, limiting their sense and enactment of personal power within different aspects of their lives. It is my hope that this research will help to uncover the sources of hegemonic behaviors within mathematics and the teaching and learning of mathematics so that ways of addressing and eliminating them might be found.

**Epiphanies.**

Many of the researchers and authors who have written about auto/ethnography reference the importance of "epiphanies" within one's biography (for example, Adams, et. al., 2014; Denzin, 2013; Ellis, et.al., 2011), those moments in the researcher's life that have resulted in conflict and possibly significant change. It is these epiphanies that cause one to reflect upon and explore aspects about our selves and others that might have otherwise gone unnoticed. As my analysis will show, my story contains many epiphanies, some of which reoccur in different forms with increasing intensity throughout.

In summary, as Roth (2005a) explains,

> The stories ethnographers create are as much a reflection of their own cultural

> positioning as they are descriptions of the positioning of others. Making these historically constituted positions clear to the reader, that is, writing auto/biography and auto/ethnography, is one way of understanding and incorporating our prejudices into our practices and into what we produce. Making sense and use of representations of some Other involves our own positioning in relation to what we are seeing as much as any meaning inherent in the images themselves; auto/biography is one of the central means of making this positioning salient. (p. 14)

Thus, with the telling of my story, I have begun my auto/ethnographic research and analysis. Moreover, the theoretical worldviews framework that I have described will become the prominent lenses through which I will read and analyze that story (and other data that will emerge later). In doing so, I am seeking understanding of the political, socially-just or unjust, and socially conscious and unconscious acts that are a result of the kinds of knowledge and ways of knowing that are being valued, devalued, or ignored.

**Questioning the unquestionable.**

Pereira, Settelmaier, and Taylor (2005) contend:

> Through auto/biographical inquiry, we might start to question that which seems unquestionable to us, a given fact, something that 'has always been there.' We might begin to confront what the phenomenologists call our 'natural attitude,' that is, our everyday way of thinking and valuing whose naturalness makes this process invisible to us in much the same way that the fish is unaware of the water in which it exists. (p. 54)

I therefore choose to enter into auto/ethnography in order to expose my thinking and valuing related to mathematics and the teaching and learning of mathematics, as well as that of those who are grounded in either the Traditional Western worldview or an Indigenous worldview. I also openly invite those who, through interest or requirement, engage with my work to do the same so that we might all come to a greater understanding of both cultures.

**Writing.**

As a final note on auto/ethnography, in the writing and documenting of this researcher, "the language should be expressive, contextualized. The text should create a virtual reality and present an aesthetic form. It should carry the author's signature and above all, it should show a degree of textual ambiguity" (Pereira, et. al., 2005, p. 57). Thus, my story is at times fanciful, it is grounded in contexts, and it is true in so far as memory can carry truth. In the conducting of this research, I never attempt to hide who I am, how I am thinking, or what assumptions I am making. Instead, I share these aspects of myself in hopes of encouraging others to do likewise and thereby further clarify (while also possibly complicating) and expand understanding.

**Positioning my research within auto/ethnography.**

My research is intrinsically connected to and grounded in my story of mathematics and me, and to have not revealed my story would have felt I was denying its existence and influence. It exists, it is part of my history, and it continuously informs and influences my current (and future) thinking about mathematics and the teaching and learning of mathematics. To hide my story would be to remove myself from the picture of my research, but it is foundational to my research. My biases, my assumptions, my beliefs, and my prejudices are all found and questioned within my story, and thus it is a story that I must share. It is also provides the reader a potentially new window of opportunity to consider and reflect upon their own stories. I cannot separate my current thinking about mathematics and the teaching and learning of mathematics from my story, thus without my story my research would be incomplete. Consequently, I choose to engage in the methodology of auto/ethnography. However, there are aspects of my research for which auto/ethography alone are not sufficient as a methodology. As such, I continue in the explanation of my methodological collage by next considering Gadamerian hermeneutics and its role within my research.

**Gadamerian Hermeneutics**

Although engaging in auto/ethnography fulfills in many ways how I wish to conduct my research, the usefulness of this methodology comes into question when I go to move beyond my own story and consider other kinds of data. In particular, the auto/ethnographical analysis of my story (done in ways also consistent with Gadamer's hermeneutics) will lead me to explore aspects of mathematics and the teaching and learning of mathematics through data that comes not from myself, but from literature. I have chosen Gadamer's hermeneutical methodology as the second in my collage of methodologies because it embraces the dialogue that I seek to have between myself, the data, and the two worldviews. Moreover, as is noted throughout the discussion of Gadamer's hermeneutics that follows, these two methodologies (auto/ethnography, and Gadamerian hermeneutics) in many ways entwine and support each other in their goals, enactment, and concerns, particularly in relation to the kinds of data that I will be analyzing. With this overview of a justification for the inclusion of this next methodology, I will now present my understandings of Gadamer’s hermeneutics.

**Interpretation, communication, and the ‘trickster’.**

Moules (2002) explains: “Hermeneutics is derived from the Greek verb *hermeneuein*, which means to say or interpret; the noun *hermeneia*, which is the utterance or explication of thought; and the name *hermeneus,* which refers to the playful, mischievous, ‘trickster’ Hermes” (p. 2). In relation to this definition, Gadamerian hermeneutics is specifically concerned with the interpretation, and hence understanding, of experiences and texts. Further, as will be explained later, Gadamerian hermeneutics emphasizes the role of language within that interpretation, as it is through language that thought is communicated. Even the mischievous nature of Hermes can be found within Gadamer’s hermeneutics in its being “organized around the disruption of the clear narrative, always questioning those things that are taken for granted” (p. 3). For Gadamer, “To understand … is, in general, to grasp something (‘I get it’), to see things more clearly (say, when an obscure or ambiguous passage becomes clear), to be able to integrate a particular meaning into a larger frame” (Grondin, 2002, p. 36). In general, Gadamerian hermeneutics, through the interpretation of text, seeks to expand one’s horizon of understanding of a concept or situation.

**Dialogue and a hermeneutic circle.**

The dialogue of Gadamerian hermeneutics is often described as a hermeneutic circle, “the to-and fro- motion of any attempt at understanding, from the parts to the whole and from the whole back to the parts… [it is a] constant process that consists of the revision of anticipations of understanding in light of a better and more cogent understanding of the whole” (Grondin, 2002, p. 47). In my research, the analysis will take on this to-and fro- motion through the varying dialogues between my understanding, the two worldview understandings, and the text. It will begin with the text of my story, but will ultimately carry on into the other, non-self generated, data that I will present. As the analysis continues, the parts from each horizon interaction with the text will merge to form wholes of interpretation and understanding, that with each successive incident of analysis will change and grow to expand understanding of my research question for both the reader and I.

**Horizons of understanding.**

Central to Gadamer’s hermeneutic philosophy and methodology is his concept of horizons of understanding. Barthold (n.d.) explains:

> Just as the visual (that is, literal) horizon provides the boundaries that allow one to see, so [Gadamer's] epistemic horizon provides boundaries that make knowledge possible. Just as the literal horizon delimits one's visual field, the epistemic horizon frames one's situation in terms of what lies behind (that is, tradition, history), around (that is, present culture and society), and before (that is, expectations directed at the future) one. (Prejudice, Tradition, Authority, Horizon section, para. 4)

More specifically, Gadamer explains that each person's understanding of any word, concept, condition, or experience is based upon an interplay of a past (or historical) horizon and a present horizon.

Gadamer (1989) explains that the historical horizon of understanding for such an instance is comprised of beliefs and knowledge about which we are trying to understand. These beliefs and knowledge come from the past, a past that unconditionally extends beyond our lifetime, which is steeped in the knowledge traditions of history within culture. One's past horizon of understanding remains fixed, regardless of one's experiences, simply because it emerges from the past.

The present horizon of understanding, on the other hand, although using the past horizon as a starting point, takes into account new interpretations, and thus new understandings, as they are experienced. With each new encounter with an idea or concept, one engages in a dialogue with that experience, ascertaining how it might be interpreted, both within one's existing present horizon of understanding and outside of it. These new understandings, regardless of their perceived truth or value are integrated into one's overall horizon (past and present) of understanding. Also, through dialogue with others, one can come to new or additional interpretations brought forward by others, which are also added to one's overall horizon of understanding. This process is referred to as the fusing of horizons and is significant within what Gadamer (1989) calls "agreement" between different horizons (to be discussed later).

**Temporal positioning.**

In addition to the past and present horizons of understanding, within Gadamerian hermeneutics,

> When we try to understand ourselves, our past and our future, we do so from a constantly changing temporal position. Moreover, we do so from a temporal position affected by a history that reflects understandings other than our own. The narratives in which we are involved and which we have to understand in one way or another not only continue even as we try to understand them, but they also continue as a confluence and even conflict of different interpretations of different narratives. (Warnke, 2002, p. 81)

Coltman (1998) furthers this discussion of the temporality of history within hermeneutical understanding when he states:

> every heremenutical engagement is thoroughly conditioned and mediated by its historical circumstances and so, in a sense, is always already underway, any specific conversation (even a counter turning dialogue) must have a beginning – or at least our discussion must begin somewhere. (p. 11)

Thus, Gadamer claims that even concepts or ideas which may seem new to us, already have a historical and present horizon of understanding, limited in scope to be sure, but ready to be engaged as soon as the opportunity arises. This position of temporality is also central to Gadamer's hermeneutics because of its impact on fallibility.

**Fallibility, language, and words.**

Just as auto/ethnography recognizes the fallibility inherent to the methodology, Gadamerian hermeneutics also acknowledges that,

> Of course, understanding often fails. But it then fails to say what would need to be said. The failure of words can only be measured by what they fail to say. The unsayable is only the unsayable in light of what one would like to say, but cannot. The limits of language thus confirm – and very eloquently – the universality of language as the medium of understanding, as Gadamer sees it. (Grondin, 2002, p. 42)

In fact, Gadamer acknowledges that all understanding is contingent upon interpretation, and within interpretation "One can always find better words for what needs to be understood, more suited' applications'" (Grondin, 2002, p. 43). Thus, Gadamer's hermeneutics is again in consonance with the methodology of auto/ethnography, seeking the best interpretation possible at the given time, but at the same time acknowledging that a full or true understanding has yet to be achieved. It is in this sense that through Gadamerian hermeneutical dialogues, "meanings of words are expanded and negotiated, ultimately shimmering" (Silverman, 1991, p. 33), giving brief flashes of nearer truths and further falsehoods that combine within one's horizon of understanding as part of the methodological agreement regarding understanding of the word.

> Not only is beauty importantly related to the true for Plato and Gadamer, it is also closely related to the good. Although Gadamer never attempted to develop an ethics or a politics, his hermeneutics is both ethical and political. The basic posture of anyone in the hermeneutical situation has profound implications for ethics and politics, inasmuch as this posture requires that one always be prepared that the other may be right. The ethic of this hermeneutic is an ethic of respect and trust that calls for solidarity. (Dostal, 2002a, p 32)

**Ethical knowledge, intersubjectivity, and openness.**

Within his hermeneutical methodology, Gadamer (1989) describes the knowledge sought as "ethical knowledge." Gadamer states:

> Not only must [this] ethical knowledge deal with a constantly changing set of circumstances and not only is its application determined by a history that is, in the course of its becoming, unceasing in its demands on us. In addition, we make and remake our ethical knowledge and ourselves in these changing circumstances, in the actions we take to apply the ethical knowledge we already possess. (Warnke, 2002, p. 85)

The seeking of ethical knowledge is done for the sake of everyone who may take understanding from one's work. Consequently, Gadamer (1989) asserts:

> Both the person asking for advice and the person giving it assume that they are bound together in friendship. Only friends can advise each other or, to put it another way, only a piece of advice that is meant in a friendly way has meaning for the person advised … we discover that the person who is understanding does not know and judge as one who stands apart and unaffected but rather he thinks along with the other from the perspective of a specific bond of belonging as if he too were affected. (Gadamer, 1989, p. 323)

Thus, like auto/ethnography, Gadamerian hermeneutics is based upon the interconnectedness and intersubjectivity of those seeking understanding.

Gadamer (1989) calls the interconnectedness and intersubjectivity of his methodology "openness", explaining: "When two people understand each other, this does not mean that one person 'understands' the other… Openness to the other … involves recognizing that I myself must accept some things that are against me, even though no one else forces me to do so" (p. 361). Of course, it is important to remember that in saying acceptance, Gadamer is referring only to accepting the understandings of others into one's own horizon of understanding in recognition, without necessarily compromising one's beliefs. He is not implying that one must forfeit their entire horizon of understanding by accepting that of another; however, as Coltman (1998) notes: "openness to the other and a willingness to put one's own prejudices at risk constitute the principal modes of comporting oneself in a genuinely dialectical hermeneutic" (p. 53). By starting with the analysis of my story, which will expose my own prejudices, and by taking on the role of the Other when analyzing my research data through the lenses of the Traditional Western worldview and an Indigenous worldview, I contend that I will bring such openness to my work.

**Language.**

As alluded to earlier, language also plays a central role within Gadamerian hermeneutics:

“Understanding for Gadamer, is itself always a matter of interpretation. Understanding is also always a matter of language” (Dostal, 2002b, p. 1). In fact, Gadamer (1989) asserts “Being that can be understood is language” (p. 432). In other words, understanding is as a result of language (p. 432). Consequently, Gadamerian hermeneutics, in direct contradiction to quantitative methodologies and the Kantian view of the world, “affirms the primacy of the spoken over the written” (Dostal, 2002b, p. 2).

Grondin (2002) further explains the role of language within Gadamerian hermeneutics:

> When presenting his own ideas or analyzing concepts, Hans-Georg Gadamer likes to follow the lead of language. The fact that basic notions he is unfolding often have many very different meanings does not bother him. Quite on the contrary, he sees in this plurality of meaning an indication that language, long before thinking, is perhaps up to something essential. (p. 36)

It is important within my research that I too keep an open mind to alternate meanings for language, particularly as I will be engaging with text both through my own horizon of understanding as well as my present understandings of horizons of understanding of Others who are either grounded within the Traditional Western worldview or an Indigenous worldview. Consequently, I should not be surprised, but actually anticipate that the same text will result in different understandings through these particular analytic lenses. Even if the interpretations from the different horizons of understanding result in the same concepts, it is very likely that the horizons of understanding will interpret or respond to those concepts in the same ways.

Grondin (2002) explains: “To understand in Gadamer’s sense is to articulate (a meaning, a thing, an event) into words, words that are mine, but at the same time those of what I strive to understand.” (p. 41). The goal then of Gadamer’s hermeneutics, “is that the [interpreter] be taken up by what he seeks to understand, that he responds, interprets, searches for words or articulation and thus understands” (p. 41). Understanding in Gadamer’s hermeneutics emphasizes both intrapersonal and interpersonal, the same intersubjectivity as sought and valued within auto/ethnography.

**Authority of traditions and frameworks.**

As in the methodology of auto/ethnography, Gadamerian hermeneutics holds that “individual’s understanding occurs in larger historical and heremenutical contexts” and it “accords great importance to the role of traditions and … in any interpretation” (Dostal, 2002b, p. 3). Further, Gadamer (1989) acknowledge that traditions, and any frameworks that are brought into research necessarily exercise "authority" (p. 277) over understanding. The assumed role of traditions and prejudice within Gadamer's hermeneutics parallel those within auto/ethnography; however, his concern about frameworks claiming of authority within the creation of understanding causes me to pause given the role of the theoretical worldview framework within my research. In defense of the incorporation of this framework into my research, however, I argue that the worldviews defining it were specifically chosen, at least in part, because they presented two very different sets of values relating to knowledge and ways of knowing. Consequently, throughout the analysis of my research, each worldview will take turns assuming the authority. It is my intention to allow each worldview to speak from its own perspectives both to and about the text without interference, influence, or prejudice from the other worldview. I therefore contend that in this instance, since the framework is not based upon a hierarchy of authority, the implementation of it need not lead to a corruption of understanding through an assumption of authority.

Like Moules (2002), I recognize that "I cannot remove my subjectivity from my work, but I can take it up with a sense of responsibility in recognizing how it translates into the way I listen to my participants, what I hear, what stands out to me, and how I interpret it" (p. 12). Gadamer (1989) also contends that within dialogue: "The important thing is to be aware of one's own bias, so that the text can present itself in all its otherness and thus assert its own truth against one's own fore-meanings" (p. 269), which my use of auto/ethnography is meant to achieve. Another connection between Gadamerian hermeneutics and auto/ethnography is in relation to the role of self: "to understand always implies an element of self-understanding, self-implication, in the sense that it is always a possibility of my own self that is played out in understanding" (Grondin, 2002, p. 38). Therefore, the inclusion and exploration of "I" within both methodologies are complementary, demonstrating how auto/ethnography and Gadamerian hermeneutics might be used as co-methodologies.

**Practical application of knowledge.**

Within his hermeneutic methodology, Gadamer also posits:

> that the practical application of knowledge is inherent in the very understanding of something. Practical application is not, on Gadamer's account, an external, after the fact, use of understanding that is somehow independent of understanding. All understanding is practical. (Dostal, 2002b, p.3)

In other words, " one who 'understands' something is not so much someone endowed with a specific knowledge, but someone who can exercise a practical skill" (Grondin, 2002, p. 37). Within my research then, it is through the inclusion of Gadamerian hermeneutics within my methodological collage that I am therefore able to speak to possible actions that might be taken, applications that might be possible from the understanding gained through my research.

**Agreement.**

Understanding within Gadamerian hermeneutics "can also mean in German 'to agree,' 'to come to an agreement,' 'to concur'" (Grondin, 2002, p. 39). The agreement Gadamer is addressing, however, should not be confused with having attained true understanding or interpretation of the text. Rather, it is the acknowledgement of the fusion of horizons of understanding. All those involved in the dialogues of Gadamerian hermeneutics are not expected to come to consensus or acquiesce to one person's horizon. Instead, the intention is to bring to light as many different understandings as possible within a single horizon that everyone shares, regardless of whether they all accept the same parts of that horizon as their individual truths.

**Finitude.**

Within his hermeneutical methodology, Gadamer (1989) also acknowledges that all humans are finite, that is, that it is not possible for a human to understand everything. However, Gadamer also speaks to the "finitude" of human knowledge, which "points to a dependency of knowledge on conditions that the human knower can never fully know. And if these conditions cannot be fully known, then this challenges us to revise our understanding of the type of autonomous control we can hope to exercise over our own cognitive endeavors" (Wachterhauser, 2002, p. 57). However, Gadamer does not only associate this finitude of human knowledge as a deficit within his hermeneutics. Rather, he argues that it is in relation to all methodologies and methods of research. Like the revealing and analyzing of Self in auto/ethnography, Gadamerian hermeneutics discloses rather than denies its limitations.

**Positioning of my research within Gadamer's hermeneutic methodology.**

Gadamer (1989) argues that in engaging in his hermeneutic methodology, "is substantively driven rather than methodologically given … it is not possible to determine a way to proceed without being guided by the topic" (Moules, 2002, p. 13). It was my story, and its temporal culmination with Leroy Little Bear's (2000) exploration of the values of Western and Indigenous cultures that urged me towards this research. In so doing, the story became a necessary part of my research, bringing me to auto/ethnography. However, how to engage with my story, and how to move beyond it within my research, brought me to Gadamer's hermeneutics (as well as the soon to be discussed grounded theory). Gadamer (1989) declares that hermeneutic inquiry begins with an experience of being addressed by a topic, and in my research it does so as the process behind my use of auto/ethnography. Although there is at least one subtle difference between Gadamer's hermeneutics and auto/ethnography, in that in the latter methodology the "practitioners sometimes return to the participant for member checks to authenticate and substantiate how well they were represented" (Moules, 2002, p. 15). Because my research only involves one real participant, myself as I am the one who will be engaging in the dialogue on behalf of the two worldviews, this step in the auto/ethnographic and methodology is of no consequence anyway. With no other notable discrepancies between the two methodologies, I contend that the many ways in which I have detailed about how Gadamer's hermeneutics and auto/ethnography align and support one another make the use of both methodologies within my collage of appropriate and effective.

**Grounded Theory: Methodology**

While the methodologies of auto/ethnography and Gadamer's hermeneutics form a strong foundation for the researching of my story in relation to the question of what kinds of knowledge and ways of knowing are valued in mathematics and the teaching and learning of mathematics, a third methodology is needed within my methodological collage that will allow me to move my research beyond my story. In my research I also want to move into less personal, and more literature-based data. Further, as so many of the conflicts and tensions within my story have yet to find a comfortable (at least for me) resolution, I ultimately am seeking new theory to address these issues. In order to fill this need, I have chosen to also include grounded theory within my methodological collage. How grounded theory fits with and complements auto/ethnography and Gadamer's hermeneutics within my research will be made clear through the following discussion of grounded theory.

**Grounded theory in general.**

Heath and Cowley (2004) explain: "Fundamental to grounded theory is the belief that knowledge may be increased by generating new theories rather than analysing data within existing ones" (p. 142). In other words, the function of grounded theory is to analyze data for the purposes of generating new theory, which can then (later) be applied to other data, rather than continuing to try to use existing theories that have not proven sufficient in explaining and resolving tensions and problems found within contexts and their data. Importantly, grounded theory may be turned to when the researcher "is already aware that there is a lack of knowledge" (Heath, & Cowley, 2004, p. 143). It is precisely this lack of (sufficient) knowledge present within literature and theory to address the epiphanies within my story that are most concerning to me that has (at least in part) led me to the selection of grounded theory as my third methodological approach.

Grounded theory most notably came to the attention of researchers, and in particular qualitative researchers, with the release of Glaser and Strauss' (1967) *The Discovery of Grounded Theory: Strategies for Qualitative Research.* What the authors proposed was a new methodology (as well as an accompanying method) for "the discovery of theory from data" (Glaser & Strauss, 1967, The Discovery of Grounded Theory section, para. 1). More specifically, Glaser & Strauss explained:

> Generating a theory from data means that most hypotheses and concepts not only come from the data, but are systematically worked out in relation to the data during the course of the research. *Generating a theory involves a process of research.* (Grounded Theory section, para. 8)

Traditionally, within most research methodologies and methods, new theory results from a researcher's hypothesis, followed by the collection of data related to the testing of that hypothesis. In grounded theory, on the other hand, the researcher does not start with a hypothesis, only an interest or curiosity – the researcher starts with a research question only.

Within grounded theory, "the research process itself guides the researcher towards examining all of the possibly rewarding avenues to understanding. This is why the research method is one of discovery and one which grounds a theory in reality" (Corbin & Strauss, 1990, p. 6). It should be noted that, just as within Gadamerian hermeneutics and auto/ethnography, grounded theory also emphasizes determining all possibilities that can be identified through the research process.

**Where it begins.**

Flick, (2014) notes that "there can be several starting points for a grounded theory study ... researchers' curiosity... personal experience or concern, gaps in the state of a scientific field... the emergence of a new phenomenon or the discovery of a new problem" (p. 399). As my research begins with my personal experiences as shared through my story, and is seeking understanding about aspects of mathematics and the teaching and learning of mathematics which are not yet understood and problems not yet resolved, this makes grounded theory an ideal choice for inclusion in my methodological collage. In addition, “grounded theory sees researchers as social beings whose experiences, ideas and assumptions can contribute to their understanding of social processes observed” (Heath, & Cowley, 2004.p. 143). In the case of my research, personal experience and concern, as found within my story, are the impetuses for engaging in grounded theory (as well as auto/ethnographic and Gadamerian hermeneutic) research.

**Seeking understanding.**

Within grounded theory “one enters the field open to realising new meaning and, via cycles of data gathering and analysis, progressively focuses on a core problem around which other factors will be integrated” (Heath, & Cowley, 2004,p. 143). Like Gadamerian hermeneutics then, grounded theory is focused on creating and expanding understanding. Moreover, the two methodologies are similar in that they both acknowledge the interplay between data (text) and the researcher. Within Gadamer’s hermeneutics, this interplay is the back and forth (part to whole) dialogue between the researcher and text, while in grounded theory the part to whole process is supported through the introduction of new data regarding influencing factors. The possibility to merge these two methodologies lies in engaging in research that involves a hermeneutic dialogue with an expanding of sets of data which are sought as a consequences of the hermeneutic dialogues.

**Conceptual labels.**

In grounded theory, “incidents, events, and happenings are taken as, or analyzed as, potential indicators of phenomena, which are thereby given conceptual labels” (Corbin & Strauss, 1990, p. 7). Thus, when analyzing a set of data, the researcher seeks to identify possible relationships in terms of actions and meanings between the details of the phenomena and their research question, and then those relationships are noted and labeled accordingly (Charmaz, 2012). As the conceptual labels emerge through analysis, the researcher then collects further data related to the concepts that have been labeled. The purpose of collecting additional data is to either confirm or invalidate the labeled concept in relation to the research question. Throughout the iterations of data collection and analysis, the researcher is making “constant comparisons” (Corbin, & Strauss, 1990, p. 9) to ensure at least the reduction of bias in the researcher’s analyses. Any conceptual labels that do not reappear or contradict evidence within new data are not researched further.

**Conceptual categories.**

As the iterative process of collecting and analyzing new data continues, "one generates conceptual categories or their properties from evidence; then the evidence from which the category emerged is used to illustrate the concept. The evidence may not necessarily be accurate beyond a doubt (nor is it even in studies concerned with only accuracy), but the concept is undoubtedly a relevant theoretical abstraction about what is going on in the area studied" (Glaser & Strauss, 1967, Accurate Evidence section, para. 2). An important part of grounded theory is that the conceptual labels, and ultimately the conceptual categories are not pre-determined; rather, they emerge from the analysis of the data, in response to the question asked. When the collection of data and analysis for conceptual labels and categories no longer produce new insights, the researcher is said to have reached "theoretical saturation" (Flick, 2014, p. 403) or "conceptual … 'density'" (Strauss & Corbin, 1998, p. 161), and the researcher decides that there is no point to collect any further data. At this point, the researcher looks to either integrate categories that come from the conceptual labels in order to describe an emerging theory (Strauss, & Corbin, 1988) or in the process of refining the conceptual labels into categories, integrating them into an emerging core which becomes the theory (Glaser, 2004).

**The divergence of Glaser and Strauss.**

Although somewhat foreshadowed within their initial work, over time Glaser and Strauss have diverged in their approaches to grounded theory, particularly in relation to the role of literature

> Glaser and Strauss both acknowledge that the researcher will not enter the field free from ideas, but differ considerably in the role they see for the literature. Discovery is at the heart of both researchers' ideas; one enters the field open to realising new meaning and, via cycles of data gathering and analysis, progressively focuses on a core problem around which other factors will be integrated" (Heath & Cowley, 2004, p. 143).

Glaser argues for the researcher to have an overview of the literature related to the research question and its source; however, he cautions that in-depth literature based knowledge can desensitize one to the data. Instead, he argues: "More focused reading [should] only [occur] when emergent theory is sufficiently developed to allow the literature to be used as additional data" (Heath & Cowley, 2004, p. 143). Strauss, on the other hand, argues: "while diffuse understandings provide sensitivity, both specific understandings from past experience and literature maybe used to stimulate theoretical sensitivity and generate hypotheses" (Heath & Cowley, 2004, p. 143).

Given the importance of my story, and thus my past, as well as my intended use of literature as data within this research, it would be prudent that I assume that my use of grounded theory within this research be aligned with Strauss. This is more an alignment of necessity and convenience than necessarily one of conviction, as I also see merit to the claims made by Glaser. It is because of the dominant role that literature plays as informant to my research (i.e., as data) that I necessarily will have to be engaging with literature deeply throughout my study. The use of literature as a data source is supported by Corbin & Strauss (1990): "the data for a grounded theory can come from various sources. The data collection procedures involve interviews and observations as well as such other sources as government documents, video tapes, newspapers, letters, and books--anything that may shed light on questions under study" (p. 5), which gives wide berth to the researcher in seeking appropriate and meaningful data to analyze.

With the above understandings of the methodology of grounded theory, and of how it serves to complete the methodological collage for my researcher, I must now turn to my methods, to how I plan to use this triad to conduct the research. Of the three methodologies, grounded theory has the most explicit delineation of methods of research in relation to the methodology, and so I return to grounded theory in my explanation of my methods.

**Grounded Theory: Method**

In turning to grounded theory to define and justify the methods I will be using in my research, I return to an earlier statement in which I specified that I would be specifically engaging in Strauss' approach to grounded theory. Thus it is to Strauss (and Corbin) that I also turn to for my detailing of my methods. In so doing, I will first provide a brief explanation of Strauss' methods, after which I will provide more specific details of how implementing those methods will occur within my research.

**Codes, categories and constant comparing.**

In grounded theory, how one codes the data is foundational to working towards the development of theory. When analyzing the data, "The incidents, events, and happenings are taken as, or analyzed as, potential indicators of phenomena, which are thereby given conceptual labels. (Corbin & Strauss, 1990, p. 7). As the data is coded, the incidents and the conceptual labeling are constantly compared in order to "accumulate the basic units of theory. In the grounded theory approach such concepts become … more abstract as the analysis continues" (Corbin & Strauss, 1990, p. 7). As the analysis of new data continues, the researcher also analyzes the relationships between concepts, as "Concepts that pertain to the same phenomenon may be grouped to form categories" (Corbin and Strauss, 1990, p. 7). Naturally, not all concepts should be expected to fit into categories, and the categories that do emerge will be more abstract in nature.

These "Categories are the 'cornerstones' of a developing theory. They provide the means by which a theory can be integrated" (Corbin and Strauss, 1990, p. 7). Corbin and Strauss warn, however, that the simple grouping of concepts under a more abstract title does not make that grouping a category: "To achieve that status … a more abstract concept must be developed in terms of its properties and dimensions of the phenomenon it represents, conditions which give rise to it, the action/interaction by which it is expressed, and the consequences it produces" (Corbin & Strauss, 1990, p. 7-8). It is the goal of grounded theory that, over time (i.e., successive data collection, analysis, and comparisons), the categories will merge into a theory.

**Data collection and sampling.**

As noted, grounded theory requires ongoing collection of data ("sampling"); however, sampling is not done "in terms of drawing samples of specific groups of individuals, units of time, and so on, but in terms of concepts, their properties, dimensions, and variations (Corbin & Strauss, 1990, p. 8). That is to say, when new data is collected it is done so because it is believed to connect to the concepts (and categories) that have emerged so far.

**The analysis cycle.**

Constant comparisons between incidents within the data, concepts, and categories "assists the researcher in guarding against bias" (Corbin & Strauss, 1990, p. 9) by consistently checking if the new data supports the concepts and categories that have been emerging. Thus, the general method for conducting grounded theory research can be described as:

1. analyzing initial data for concepts (open coding)
2. collecting new data to verify the concepts
3. analyzing the new data for concept validation
4. analyzing of concepts for relationships to determine (sub-)categories (axial coding)
5. collecting new data to verity the concepts and categories
6. analyzing the new data for concept and category validation
7. analyzing of concepts for relationships to determine categories
8. analyzing of categories to determine broader categories (axial and possibly

selective coding).

The third step in the process is repeated until no new relationships between concepts or categories can be determined and there is a strong unification of a number of categories and concepts into a single "core" category (selective coding).

> The core category represents the central phenomenon of the study. It is identified by asking questions such as: What is the main analytic idea presented in this research? If my findings are to be conceptualized in a few sentences, what do I say? What does all the action/interaction seem to be about? (Corbin & Strauss, 1990, p. 14)

Once a core category has been identified it becomes the foundation of the new theory proposed via the methodology of grounded theory. Thus, this theory emerges from a "detailed and dense process" that is "fully described" (Heath, & Cowley, 2004, p. 146).

An additional note should be made with respect to the coding proposed by Corbin and Strauss (1990): "A single incident is not a sufficient basis to discard or verify a hypothesis. To be verified (that is, regarded as increasingly plausible) a hypothesis must be indicated by the data over and over again" (p. 13). This tells the researcher that in order for a concept to become a part of a category, and a category to become part of the final theory, it is necessary that there must be consistent and abundant validation of the concept or category.

**Positioning research within grounded theory methodology and methods.**

As previously noted, the addition of grounded theory within my methodological collage provides me with an avenue for expanding outward from my auto/ethnographic consideration of my story to other sources of data. At the same time, my choice of additional sources of data will be informed by my auto/ analysis of my story. Moreover, Gadamer's hermeneutical methodology provides me the format through which I will be carrying out the continuous coding and constant comparing of data, concepts, and categories of grounded theory, that of engaging in dialogues between the data and the two worldviews (the Traditional Western and an Indigenous). All three methodologies are also in agreement with respect to the often-contentious issues of the inclusion of self, prejudice, and intersubjectivity. In this research, I will begin with an auto/ethnographic analysis of the story of *Mathematics and Me* through a Gadamerian hermeneutical dialogue involving the worldviews of the framework and using the methods of Glaser's grounded theory. This analysis will be followed by similar dialogues and analysis of additional data advocated for by my auto/ethnographic positioning and gathered and analyzed (hermeneutically) through grounded theory. Thus, the methodological collage that I will be engaging in is a purposeful and directed interplay between the three methodologies, giving my research and it's analysis cohesion and purpose.

## My Research Methods

Taking the lead from this general understanding of the method of engaging in grounded theory as proposed by Corbin & Strauss (1990), I will now explain how I will enact and document these processes within my research. As noted before, my starting set of data is my story (the auto/ethnographic contribution to my research).

### Coding of my story.

As previously noted, I will be analyzing the data in my story from the perspective of two different lenses (that of the Traditional Western worldview, and that of an Indigenous worldview), and in particular I will be looking for indicators of phenomenon within the data that may indicate, relate to, or influence, the kinds of knowledge and ways of knowing that are or are not being valued. Upon identification of any such phenomena, I will use open coding to label the concept it represents, and record in my notes both that concept and a description of characteristics pertaining it found within the data. As the coding of my story progresses, I will continue to add new concepts as they present themselves, as well make edits to descriptions to reflect my growing understanding of the concepts.

The coding of my story may also result in some axial coding, in which concepts identified within previous parts of my story may be seen to relate to or even merge together. In such cases, I will be identifying the new category that results, as well as documenting its defining characteristics as seen within the data.

### Data sampling and continued open and axial coding.

Upon completion of this initial data analysis, I will then use my story and its analysis to determine other kinds of data that I feel would contribute to my overall research and speak to the particular concepts and descriptions I have identified thus far. Repeating the analysis process that I used for my story, I will be either verifying, questioning, or creating additional concepts. In addition, I will be analyzing my concepts to see if broader categories are emerging that relate and include any number of those concepts. I will repeat this process of presenting and analyzing new data until I find that many concepts are merging into categories and that at least some of those categories are merging into a core category. At that point, I hope to be able to propose a new theory that speaks to the kinds of knowledge and ways of knowing that are, could be, and should be valued within mathematics and the teaching and learning of mathematics, along with the possible consequences of actualizing that theory.

**Saturation.**

When my coding of the data is no longer providing new concepts and the creation of categories has come to a point where a core category (or core categories) is emerging, the coding will have become saturated and I will end the sampling of new data and its analysis. At that point, I will turn my attention to the proposing of a new theory based upon the core category/ categories that have emerged.

**Documentation of my research.**

The presentation of this part of my research will need to follow the flow of my engagement with it, as did my story and the theoretical worldview – they were presented first because they are where my research and my research question came from. Therefore, I will next provide the analysis of my story, as the data is already present. Following that analysis, I will be documenting the cycle of sampling and coding as it occurs, hopefully ending the cycle through the proposal of a new theory. Within each analysis section, the reader will be provided with a summary of the prominent features of the particular data set that speak to the valuing of kinds of knowledge and ways of knowing, the response of the two worldview positionings with respect to those prominent features, and finally an explanation of the coding (open and axial) that is implied by those responses.

**Personification of the worldviews.**

One final note needs to be made regarding my use of the theoretical worldview framework in the analysis of the data. In order to cut back on wordiness, I will often make statements such as "an Indigenous worldview would see this incident as…." Of course, neither an Indigenous worldview nor the Traditional Western worldview "sees" or "views" anything, but someone holding such a worldview does. In so writing that the worldviews "see" or "view" a particular incident or context in a certain way, I am, in fact, asking the reader to think of the worldview in terms of a person who holds that named worldview.

**Back to the beginning.**

With my methodological collage and methods explained, my research can now move forward into the analysis, and subsequent data collection and analysis, sections. Having come this far, I now take the reader back to the beginning – back to my story.

## Analysis of My Story

At this point in my analysis, I call upon all three methodologies within my collage. I bring forward auto/ethnography as I read and analyze my personal story for its epiphanies related to the cultures of mathematics and the teaching and learning of mathematics; I engage in Gadamer's hermeneutics, as I use dialogue between my story and each of the two worldviews to obtain greater understanding of these two cultures; and, I use grounded theory coding and comparisons to label incidents that provide indications of concepts inherent within the story, as illuminated by the worldviews, that speak to the valuing of mathematics and the teaching and learning of mathematics. As each part of my story highlights awakenings, shifts, or outright changes in my thinking about mathematics and the teaching and learning of mathematics, I consider each part separately within this analysis.

### "Long before I ever entered school" Analysis

In this introductory section of my story, the reader is provided a very general overview of how, as a young child, I thought about mathematics. In particular, I focus on how mathematics and story was categorized within my mind.

#### Prominent features within the data.

In the introduction, the emphasis is on my separating of mathematical knowledge from story-based knowledge, demonstrating my belief that one type of knowledge should not interfere with the enactment of another. My view was that mathematics and story were two very different kinds of knowledge and ways of knowing, each of value on their own, but not of value to each other or with each other. Thus, mathematics and story were strongly categorized, isolated, and even abstracted in my earliest memories.

#### Dialogue with the Traditional Western worldview.

When considered through the lens of the Traditional Western worldview, my beliefs about mathematics and story are in agreement with the worldview's emphasis on valuable knowledge being categorized and isolated. As well, someone grounded within the Traditional Western worldview would likely consider my view of the distinction between mathematics and story, and what can be known about and through them, as a valuable way of knowing.

#### Dialogue with an Indigenous worldview.

Alternatively, when considered from the perspective of an Indigenous worldview, my rigidity in the separation and categorization of the two kinds of knowledge would be seen as counter-productive to the seeking of knowledge for the purpose of constructing, maintaining, and expanding relationships. Consequently, the knowledge that I held in relation to either mathematics or story would be seen as possibly being of less value because I have actively not sought connections between them. Further, because of an Indigenous worldview's acceptance (and seeking) of diversity in ways of knowing, my stance on the isolation of mathematics and story would also be deemed as not as valuable as it could be.

**Coding and explanation.**

Overall, there is a single concept label that I have assigned to the introduction to my story, specifically, categorization and isolation. Associated with this concept, I have noted that, depending upon one's worldview, dealing with knowledge in a categorizing or isolating manner may be perceived as either an advantage or a limitation. In particular, my view of mathematics at that time is one that would be seen as an advantage within the Traditional Western worldview, yet a limitation within an Indigenous worldview. From this first moment in my story, the first epiphany, and through the engaging in dialogue between my description of that epiphany and each of the worldviews, the first concept has emerged from my data: categorization and isolation.

**"My first mathematical memory" Analysis**

This part of the analysis of my story focuses on my time before entering school. In particular, it focuses on the common activities that I engaged in as a child. In this part of my story, there are two kinds of epiphany events: one defined by my responses to television, and the other in response to activities I was engaged in outside of the TV programs in question.

**Prominent features within the data.**

My need for the separation of different kinds of knowledge continues in my memories of the TV shows that I did or did not like to watch. Those shows that I liked, such as *Kingo Bing*o and *Mr. Dress-up*, had, for me, a defined knowledge purpose (mathematics and story, respectively). From my perspective these shows did not cross the knowledge boundaries that I believed in. I also preferred those shows that moved me beyond what I already knew, creating a sense of a learning trajectory for me.

Interestingly, in more recent discussions with friends, I was at first surprised to hear them speak in detail about the host of *Kingo Bingo,* and aspects of *The Friendly Giant* and *Mr. Dressup* that were strongly related to the learning of mathematics. Although I have memories of the aspects of which my friends speak, such as common things the *Kingo Bingo* host would say, or particular episodes of *The Friendly Giant* or *Mr. Dressup*, I did not recognize, at the time of my story, that this was an infringement upon my need for knowledge separation. Essentially, I appear to have blocked these features of the programs from the memories that are most clearly defined to me. It was not that I was not aware of the other things happening in each program, but determined that I knew the sole purpose of a program I was only interested in that aspect of the program.

From this perspective, it is no wonder that I disliked *Sesame Street* at that time. For whatever reason, I had decided that the show's purpose was mathematical in nature, so when it did not challenge me mathematically, I decided it had no real value. Likewise, when I think now on my engagement with *Kingo Bingo,* I realize that there was so much more than just the identifying of the numbers called with those on the cards in front of me, even more than the personality of the host, was of value. After my Mom and I moved the piano bench, and after my parents and grandparents had picked up the bingo cards for me, my need for relationships with others while working with the numbers was gone. It was all about my manipulation of the numbers – my specialization in my knowledge of the Natural numbers. I have no immediate recall of the bingo caller or if there were other people on the show – I was completely focused on the numbers in isolation of all else. Just as auto/ethnography aims to do, this part of my story has illuminated my propensity to isolate and have singular focus on one type of knowledge at a time.

My short recounting of playing *Sorry* with my grandfather is very much analogous to that of bread making in its emphasis on the categorization and isolation of knowledge. The game, to me, was all about numbers – rolling numbers and counting numbers. It was not about sound effects or silliness; it was serious mathematical business. This part of my story also highlights my valuing of absolute truth and the correct way. Even though I knew my grandfather was teasing me, trying to play with me in a non-mathematical way, I was annoyed with him because number is not something to be played with. What a number is is fixed and indisputable. Even the twinkle in my grandfather's eye as he double counted a square or as he animated his moves with sound effects was not enough to shake my resolve. Numbers had to be right and they had to be respected because they were much more important than goofiness when playing *Sorry*. I now recall how those same sound effects, in a different situation, such as when my brothers and I played *Twister* or *Checkers,* were not a distraction or annoyance to me. They added to the experience, because the experience was, in my mind, not about mathematics, it was about playing and relationships.

Similarly, although I now tend to focus on and value aspects of my mother's making of bread beyond the counting of the cups of flour such as the flour dust puffing out of the bowl, the tea towel slowly pushing upwards from the bowl as the dough rose, and Mom and I pounding down the inflated dough, only to set it to rise again, at the time what was most important was the mathematics involved. Making bread was an excursion into mathematics. Everything else about the experience sat of the periphery of importance. Interestingly, from my older perspective and broader knowledge of mathematics, I now realize how even my enthusiasm for engaging in the mathematics of the experience was even more isolated than it first appears, as I failed to acknowledge or value the geometrical and measurement knowledge that is also housed within the event. I knew about shapes and I new about measuring, but my complete focus was on the number, thus again reflecting my tendency to abstract experiences into a single focus determined by a hierarchy of knowledge that I valued. It is quite possible that mathematics of quantity, rather than spatial mathematics, was positioned higher within this hierarchy because I viewed the spatial aspects of the experience part of the story as not as important as the mathematics.

**Dialogue with the Traditional Western worldview.**

Thus, just as was the case for the introduction to my story, when viewed through the lens of the Traditional Western worldview, this part of my story is supported by the worldview's valuing of knowledge that is categorized and isolated. Moreover, my persistence in associating various TV programs with particular kinds of knowledge (in this case, mathematics or story), whether subconsciously intentional or an oversight, also resonates slightly with the Traditional Western worldview's valuing of a knowledge within a hierarchy. Given set parameters, based upon my decision regarding the purpose of each of the programs, certain types of knowledge were, in my mind, more important (and valuable) than others, and those programs that did not align with this hierarchy of knowledge were programs that I deemed not worthy of watching. Even my method of establishing these hierarchies of knowledge, which resulted from my condensing of an entire program into a single kind of knowledge repository, aligns with the Traditional Western worldview.

Further, from the perspective of the Traditional Western worldview, how I responded to playing *Sorry* with my grandfather would also be seen as engaging with valuable knowledge and disregarding knowledge of no value. My irritation with my grandfather was also my way of telling him that his silliness and sound effects were not valuable within the game. There was one important focus in the playing of *Sorry,* a mathematical focus, and that focus was needed to achieve my goal – to win. Mine was the right way to play *Sorry*, a stance that would be highly regarded by someone grounded within the Traditional Western worldview.

Thus, whether it was in the way I responded to TV programs, or how I engaged in activities with friends and family, my focus was consistently one-sided. In these particular cases, either number (and mathematics) was what I valued in the moment, or story and relationships was what I valued, but never both. As such, my approach to both types of endeavours was to be grounded in a single, isolated phenomenon that I had identified as most important in the situation. For me, there was a purpose for everything I engaged in, and that purpose was singular in nature, which is indicative of thinking grounded within the Traditional Western Worldview and it's valuing of categorization, abstraction, isolation, and singularity.

**Dialogue with an Indigenous worldview.**

Conversely, from the perspective a person grounded within an Indigenous worldview, my decisions regarding what programs were worth watching and why would be subject to scrutiny on many fronts. As previously noted in my analysis of the introduction to my story, my need to categorize and isolate different kinds of knowledge would not be considered especially advantageous from such a person's viewpoint. Further, my thinking about knowledge in terms of hierarchies of importance would also be seen as somewhat questionable, or limiting, within an Indigenous worldview, as it would not allow me the flexibility to recognize or value kinds of knowledge that were not deemed valuable within my hierarchy. Likewise, my engagement with TV programs, which may include complex relationships and understandings, in abstracted ways in which I only seek a particular kind of knowledge, would be seen restricting of the knowledge that I could gain.

With respect to the non-TV related activities, my thinking would not have been rejected from the perspective of an Indigenous Worldview, but my refusal (or inability) to reconcile and relate different kinds of knowledge and ways of knowing would be viewed as limiting. If my thinking had been grounded within an Indigenous Worldview, I would have actively sought and established relationships between the types of knowledge and ways of knowing within all of the encounters that I recalled in my story. Thus, although within an Indigenous Worldview my stance on isolating knowledge would have had a place (based upon the context in which it occurred) the rigidity of my positioning at this young age would have been challenged if I was trying to be grounded within an Indigenous Worldview.

**Coding and explanation.**

According to the methodology of grounded theory, I found that within the analysis of this section of my story, the previously identified concept of categorization and isolation is again present and the same relationship between worldview and this concept, that of being advantageous or limiting (the Traditional Western worldview and an Indigenous worldview, respectively) has been upheld. The hermeneutic dialogues between the two worldviews and the epiphanies throughout have also given rise to additional concepts. First, is the concept of hierarchy. The hierarchy that this part of my story assigns to different kinds of knowledge is one that is in agreement with the Traditional Western worldview, while within an Indigenous worldview creation of such a hierarchy would have been accepted, but questioned in terms of how it might not result in the most valuable of knowledge.

Abstraction also emerged as a concept when this part of my story was viewed through both of the two worldviews, and as is true of the concepts of categorization and isolation, and hierarchy, it sits comfortably within the Traditional Western worldview, but would be seen as overly narrow from the perspective of an Indigenous worldview. Again, an Indigenous worldview would not reject the abstraction, but it would question the value of doing so when other possibilities have not been considered or included.

Finally, this section has introduced a concept, which I have labeled singularity, related to the notion of "the right way" of doing mathematics. This concept is one that is foundational within the Traditional Western worldview in its valuing of knowledge being singular (or absolute) in nature and that singularity of knowledge is accompanied by a singular correct way of knowing. An Indigenous worldview, however, does not seek singularity of knowledge or a singular way of knowing; instead, it values diversity of knowledge and ways of knowing. Within a specific context, the concept of singularity might be valuable from an Indigenous worldview; however, since I did not give any consideration to other ways of playing *Sorry*, I could not, from the perspective of an Indigenous worldview, argue that in this particular case the singularity that I was expecting and demanding was most valuable.

**"When I started school" Analysis**

This part of my story that focuses on my time in school from grades 1 to 12 (I did not attend kindergarten). The events and epiphanies that most stand out (from an auto/ethnographical perspective) includes the continued separation of story and mathematics, in fact, of all subject areas; my experience when attempting to help my friend; my encounters with doing mathematics the wrong way; and my encounters with rules.

**Prominent features within the data.**

Within these 12 years, all of my learning was separated into subject areas, taught in detachment from each other. Although I had no other way of learning to compare it to, I was successful and thus did not bother to consider alternatives.

This part of my story is also full of examples of when I struggled with not following the rules of mathematics. My first encounter with these rules occurred when I unintentionally challenged the rule of "the right way to do mathematics," a rule, that despite my own application of it earlier on, came as a surprise to me. It was while attempting to help my friend understand how to do something in mathematics that my teacher brought this rule to my attention. Although my first reaction was to feel bad because it meant my friend was continuing to struggle, I also accepted the rule as necessary. Likewise, when I was learning to add and subtract decimals, I did not question the "right way to do it" rule. Instead, I followed it religiously, although not because to do the addition and subtraction differently was to break a "cardinal sin"; instead, I followed the rule because "it worked" for me. This absolute commitment to the "right way rule" can also be seen in my experience when solving quadratic equations and the placement of the +/- sign. I did not understand the rule, but I did my best to remember it and follow it, because it was the "right way".

My other encounters with doing mathematics the "right way" were not so straightforward. In the cases of finding the answers for 8 + 5 and 8 x 7, "the right way" (memorization) had failed me (although I saw it as I had failed it), and it was only by breaking the rule that I was able to get the right answer. The torment I experienced within myself as a result of having to break the rule, to do the mathematics in a wrong way in order to get the right answer, filled me with guilt every time I did it. Eventually, the answers did become memorized, but I still knew how I had come to know them and consequently the guilt continued for a very long time.

My final experience with the "right way" rule (which I recall) was like a middle road experience for me in which I continued to always add columns of numbers in the right way for my teacher, but on the side I played with, and checked my work using, Dad's wrong way. In this instance, because I was able to carry out the "right way," I did not experience the same kind of guilt when I did my work the wrong way; however, I did question whether it was always a good use of my time.

**Dialogue with the Traditional Western worldview.**

As has been true up to this point, most of this part of my story is in strong alignment with the Traditional Western worldview. From this perspective, my emphasis on and desire for adherence to doing the mathematics as my teacher had taught me conforms to the valued way of knowing through a specialist, and the rules that I memorized and diligently tried to follow would be seen as contributing to my singular correct knowledge. A person grounded within the Traditional Western worldview would, however, likely view my inability to just memorize all of my addition and multiplication of facts as a weakness within my ways of knowing and in the knowledge that resulted.

Interestingly, my secret practicing of my Dad's method for adding columns of numbers, and using it to verify my application of the teacher's rule, again brings in the notion of hierarchies, but this time hierarchies of specialists. The notion of a hierarchy of specialization is one that is valued within a Traditional Western worldview, but I could only speculate (which I will refrain from doing at this time) as to how someone grounded within this worldview would position my teacher and my dad within such a hierarchy.

**Dialogue with an Indigenous worldview.**

From the perspective of an Indigenous worldview there would actually seem to be a bit closer alignment at some points in this part of my story. In particular, my devising of alternative ways to find answers to the addition and multiplication facts that I struggled with would be easily accepted as valuable, since it brought diversity in terms of my ways of knowing as well as increasing my overall knowledge. Similarly, my holding onto, and incorporating (on the side) of my Dad's strategy for adding columns of numbers would also be valued for the way in which it diversified my ways of knowing. Now I had two ways to answer this kind of problem. It is only in the examples of my unquestioning following of the rules (not to help my friend, and to put in the +/- sign in certain cases), that a person grounded within an Indigenous worldview might question the value of my knowledge and ways of knowing, since I am not seeking alternative perspectives or approaches, nor am I attempting to relate this knowledge to other knowledge. Further, by conceding to not help my friend, I would be seen as devaluing our relationship, which, according to an Indigenous worldview, the seeking of knowledge should not do.

**Coding and explanation.**

Following from the application of the processes of grounded theory, I have identified examples, which validate, and even expand, particular concepts previously discussed, as well as a new concept. First, this part of my story again brings forward the concept of hierarchy. The hierarchy of knowledge is again present in my feeling of guilt over not being able to just memorize the addition and multiplication facts; my way of knowing them is not as valuable of a way of knowing as strict memorization.

This set of experiences, however, take the concept of hierarchy a step further through the introduction of the idea of the specialist, of a hierarchy of people based upon the knowledge they have. Within my story, the teacher, and her knowledge and ways of knowing, is placed higher in that hierarchy than I am (hence, I am not to help my friend); however, no definitive conclusion can be reached from these memories as to who sits higher within the hierarchy – my teacher or my dad. Perhaps they even occupy the same level. From the perspective of the Traditional Western worldview this second implication of hierarchy would also be deemed appropriate, whereas within an Indigenous worldview such a leveling of specialists would be deemed as potentially problematic (but not overall unacceptable) as the hierarchy of knowledge was. The expansion of the concept of hierarchy to include hierarchies of specialization, is a possible example of axial coding as the concepts of hierarchy and specialization appears to be merging into a larger concept or category. However, further analysis of data is required to validate this unification.

Singularity of knowledge and ways of knowing also reemerges as a concept from this section of my story. In this case, the belief that there is a right way is often problematic for me personally, in that I either cannot do the right way, or I am starting to question its claim to singularity. The problematic nature of the concept of singularity has an interesting effect upon the relationship between the worldviews and myself in that when problems arise (i.e., when I create or explore alternative ways of knowing), the Traditional Western worldview would question my response, while an Indigenous worldview would embrace it.

It should also be noted that, although not discussed in the worldview analyses, because there is no change in the concept to be noted, categorization and isolation also is present, mostly in the background, of this section through my discussion of the separation of learning by subjects in grades 1 to 12. My response to this separation of kinds of knowledge remained the same as it had been from the very beginning – I accepted it without challenge or reflection. Thus, with respect to categorization and isolation this part of my story confirms my strong link to the Traditional Western worldview and my acceptance within an Indigenous worldview, with the caveat that holding this belief is limiting many of the valuable kinds of knowledge and ways of knowing that I might alternatively seek.

Finally, relationship emerges in this section of my story as a new concept. Here, relationships are of focus in two main ways: my relationship with my friend, and my recognition of the relationship between my dad's method of adding columns of numbers and the commutative property of addition. one is a relationship between people being influenced by a assumed rule of mathematics and the other is a relationship between myself and doing mathematics based upon a relationship with my dad. Each of these incidences of relationships are intrinsically bound in story, the story of my wanting to help my friend feel successful (and being told not to), and the story of Dad taking an interest in me, my homework, and my ways of knowing.

From the perspective of the Traditional Western worldview, these relational aspects would primarily be viewed as superfluous – extraneous to the actual (mathematical) knowledge of value. Not even my relating of the commutative property of addition to my dad's method of adding columns of numbers would be seen as necessarily important since all I needed at that point was to be able to add the numbers, and the teacher's rule told of how to do that. The only possible saving grace, from the view of the Traditional Western worldview in my recognizing this particular mathematical relationship, might be in that it might be assumed that I was engaging in seeking knowledge for the sake of knowledge. In reality, I was seeking this relational knowledge in order to justify and make sense of both ways of doing the mathematics, and in that light, a person grounded within the Traditional Western worldview would likely view this as an unnecessary pursuit that would not result in knowledge of value. Alternatively, and requiring less discussion, a person grounded within an Indigenous worldview would not only value the addition of these relationships to my mathematical knowledge and ways of knowing, but would encourage me to explore these relationships more fully and to seek new ones as well.

Thus, in performing open coding on this section of my story, the previously noted concept of categorization and isolation was again validated. The concepts of hierarchy, abstraction, singularity, and specialist were also validated as well as having their characteristics further elaborated. Further, a new code, relationships, was also identified within the data. Finally, the analysis of this section also gave rise to the first instance of axial coding, as evidence was found to link the concepts of hierarchy and singularity. As I continue to code the next sections, I will continue to look for evidence of the individual concepts of hierarchy and singularity, as well as of the broader category including hierarchy and singularity.

**"As it came time to apply for university" Analysis**

The next section in my story relates to my time as a university undergraduate student. Within this section of my story, four epiphanies of importance are presented: one related to my experiences in vector calculus, another surrounding my friend's (and fellow honours student's) crushing realization regarding the value of his mathematical knowledge and ways of knowing, the third being the professorial debate about mathematics and language, and finally, my experiences in relation to applying for entrance into education programs.

**Prominent features within the data.**

My experience in the vector calculus course was monumental in terms of my relationship with mathematics. Until this point, mathematics had been my forte. Other than the two previously noted dalliances into knowing my addition and multiplication facts in a way other than rote memorization, I had been extremely successful in mathematics, and had often wondered how it could be that some of my fellow classmates "just didn't get" math. I knew mathematics and how one comes to know mathematics and I was confident in my mathematical abilities and aptitude.

The vector calculus class led me to question everything I thought I knew about mathematics and how I knew it. The specific incident, in which I solved and erased the final exam problem over and over, highlights my complete commitment both to the right way to do things and the absolute truth of the knowledge that I had. It was not until I had the meeting with my professor that I realized that perhaps those commitments should be called into question. I was beginning to think that perhaps the right way is not always the only (or best) way to do mathematics, and that perhaps different ways of knowing and different kinds of knowledge were better suited to different situations (contexts). Further, I was beginning to think that mathematics could (and should) be learned and understood relationally, and not in separate pieces. When my professor pushed at the heel of my hand so that the angle of my student card changed, my eyes were opened to the notion that perhaps abstracted knowledge was not always the most useful or meaningful mathematical knowledge. As a result of these contemplations, I began to think about and do mathematics differently, and I started to invite others to do so as well.

My commitment to, and struggles with, making these changes with respect to the kinds of mathematical knowledge and ways of knowing that I valued, is quite evident in the event when my friend offered to help me in a later mathematics class. My question to this friend was seeking more than the mathematical knowledge that I had sought and valued before my vector calculus incident. I wanted to understand tensor products in a relational and non-abstracted way, and not just through definitions and procedures. Unfortunately for me at the time, but for my friend in the long run, the kind of knowledge that he shared with me was not a kind of knowledge that I was seeking. Further, he did not understand that my question was informed by a desire for knowledge different from rules. My loss in seeking, but not obtaining (at least at that time) a different kind of knowledge resulted in my receiving a mark lower than that of my friend (which was of no real consequence in the long run), but my friend's loss in not also seeking the kinds of mathematical knowledge and ways of knowing that I sought (within and beyond that class) had devastating results upon his future goal of becoming a mathematician.

The next epiphany of interest within this section of my story begins with my statement in the lounge of the Department of Mathematics and Statistics, "mathematics is just another language." I was surprised by how quickly the professors who were present took up this debate, as I had been sure that, like it had been for me for many years, the professors would have the right view of mathematics, that is, that mathematics and language (and hence, story) were isolated islands of knowledge. My surprise then turned to shock, and even a bit of horror, when I heard that this debate regarding mathematics and language had somehow made its way to questions of religion. Other than my elementary teacher's tongue in cheek (I believe) reference to the "cardinal" rule for adding and subtracting decimals, and perhaps some whispered sighs and prayers from fellow classmates when mathematics was mentioned, I had never experienced or considered a relationship between religion and mathematics. I was afraid of where this debate might head next, what insults might be levied in either the name of God or mathematics, and so I was relieved to be able to leave the lounge for the religion-free security of the classroom.

However, even within that safe place, I could not completely disconnect from what I had heard and experienced in the lounge, and I pondered questions regarding the truth of the statement of "God created zero and one" and consequently about what zero and one were, and why I was so unsure about all of this.

The final epiphany from this section of my story relates to my applications to different university education programs. Most striking for me is the different reactions that I received from each of the universities. For two of the universities, all that mattered was that I had a previous degree in mathematics. For a third university, my mathematics major was only considered good enough if it came with a science minor, and for the fourth university my degree in mathematics was received with great enthusiasm; however, my minor in English was deemed lacking and in need of rectifying because I had not taken one particular English class. Up until this time, I naïvely believed that acceptance into a university education program as a mathematics major would involve the same process regardless of the institution, at least in terms of expectations regarding minors. In trying to recall where this notion originated, I have come to believe that it was related to my thinking that within the "real world" if you could do mathematics, you could do anything, and I could do mathematics.

**Dialogue with the Traditional Western worldview.**

From the perspective of the Traditional Western worldview my experience in vector calculus gives rise to the possibility of an interesting dichotomization of a response. Since I was relying on the mathematical knowledge as I had been taught it, the "right way", the Traditional Western worldview would hold that my seeking of new vector calculus knowledge based upon the correctness of my existing knowledge, my struggles would have seemed, without further information, unexplainable and unfortunate. My subsequent transitioning to the seeking of understanding of mathematics through different ways of knowing, however, would have been deemed as tantamount to heresy, with the only possible saving grace being that I had one instance in which my professor, by tilting my hand, had shown me the *real* "right way".

With respect to my friend's adherence to learning mathematics in the way that it was taught to him – as a set of singular, categorized and isolated, and abstracted rules, definitions, and procedures in no way challenges the Traditional Western worldview's ideals regarding the kinds of knowledge and ways of knowing that should be valued. From the Traditional Worldview perspective, his failure to succeed would again have been unexplicable without further research.

The debate over whether mathematics is a language is also an interesting event to analyze through the lens of the Traditional Western worldview. On the one hand, such a claim obviously disregards the importance of categorizing, isolating, and abstracting knowledge, as to argue for such a claim would be to create a relationship between mathematics and language. On the other hand, the last argument that I heard, "God created zero and one, and man did the rest" is introducing a level of power and authority (resulting from a hierarchy whose pinnacle is God?). This statement is openly giving authority to mathematics, and particularly 0 and 1 within mathematics, within a hierarchy of knowledges, and it is also placing the human ability to create mathematical knowledge almost as high up within that same hierarchy. It gives humans power and authority over mathematical knowledge creation, and in so doing, also assigns power and authority to mathematical knowledge itself. Thus, from the perspective of the Traditional Western worldview, this debate is both in contradiction to the worldview's valuing of categorized, isolated, and abstract knowledge and in agreement with the worldview's valuing of hierarchical knowledge and knowers as well as of power and authority. I therefore choose to not draw a definitive conclusion regarding how a person grounded within the Traditional Western worldview would respond to this experience.

Finally, my experiences coming from applying to different education programs across Canada can also be considered from the perspective of the Traditional Western worldview. Putting aside my assumption that all entrance requirements would be the same for a moment (a "right way" and thus Traditional Western worldview way of thinking), each of the responses to my applications do seem reasonable from a Traditional Western worldview perspective. For example, the two universities who accepted me immediately because of my previous degree in mathematics would have been working from the mandate that they required mathematics majors, and their jurisdictions required high school mathematics teachers who were mathematics majors. In this way, my previous degree was providing me the power to gain automatic acceptance because my mathematical knowledge was being perceived as the most valuable (hierarchically speaking) criteria in the application.

Although the other two universities did not value my mathematical knowledge above all else, they each did have a hierarchical set of entrance requirements. In the one university, high up on the hierarchical levels of the entrance requirements was specific pairings of majors and minors. My being an English minor did not fit those entrance requirements and thus was deemed insufficient in terms of the specializations that I offered.

In the other university, my English minor, as a pairing to my major in mathematics, did not challenge the entrance requirements (or its hierarchy), although there may have been other major-minor pairings that would have done so, and thereby again implying a hierarchy in terms of the valuing of major and minor combinations. However, this university did have a hierarchy of requirements with respect to what constituted, beyond the number of courses, a minor in English, and I had not met those requirements. Of course, the requirements at any of the four universities would have been determined methodically and logically. In particular, there is evidence that the methods and logic used would have been such that it would meet with approval from a person grounded within the Traditional Western worldview.

**Dialogue with an Indigenous worldview.**

A person grounded within an Indigenous worldview would see both my vector calculus experience and my friend's abrupt failing out of a Master's program in mathematics very differently. My struggles within vector calculus would have been directly attributable to the inadequacy of the kinds of knowledge and ways of valuing that I had been taught, knew, and, up to that point, valued. My professors tilting of my hand would be seen as allowing me an alternative way to think about the mathematics, and her expectation that I would live up to my promise to relearn mathematics for understanding or else go back to majoring in English, would be viewed as supportive of an Indigenous worldview's valuing (and encouraging pursuit) of alternative kinds of knowledge and ways of knowing. Thus, what I would describe as my lessons learned out of my vector calculus experience would align strongly with the thinking about kinds of knowledge and ways of knowing that are valued within an Indigenous worldview.

When considered from the perspective of an Indigenous worldview, my friend's deeply lamentable and short experience within a Masters program in mathematics strangely aligns with the worldview. It aligns in that the lesson my friend learned, that the mathematical knowledge he had been taught, the rules, procedures, and definitions that he had memorized, was not mathematical knowledge and ways of knowing of value in and of itself. From the viewpoint of a person grounded within an Indigenous worldview, my friend's failure would be easily attributable to the lack of flexibility and diversity in the knowledge and ways of knowing that he had been taught and valued. Had my friend instead be taught mathematics in ways that valued diverse ways of knowing and kinds of knowledge, a person with an Indigenous worldview perspective would have found what he knew about mathematics, and how he knew it, to be of great value and thus use. Instead, an Indigenous worldview would find what he actually knew and how he knew it to be extremely limiting, and in the end, that is unfortunately how he also found it.

From the perspective of an Indigenous worldview, the direct and immediate acceptance by two of the universities, as they were desperate for math teachers in their jurisdictions, would be seen as responding to the particular context of the moment, an essential part of in determining the value of knowledge within this worldview. The responses from the other two universities, however, are missing the essence of the context. As the applicant, I did not know why the one university could not accept a math major, English minor combination – just that it was not acceptable. Similarly, I was not given a reason for the absolute need of the class I was missing. In both cases, because I do not know the contexts or stories behind these knowledges that are necessary for entering the universities programs, it is not possible to determine how or whether these two experiences of mine would correlate to an Indigenous worldview's values.

**Coding and explanation.**

Continuing on with a grounded theory analysis of this part of my story, I will first consider concepts that have previously emerged, beginning with hierarchies of knowledge. Notably, in my vector calculus experiences, and my experience with my friend's explanation of a tensor product, my valuing of knowledge seems to be flattening out the hierarchy that I had previously held to. That is to say, upon my commitment to relearning mathematics to my vector calculus professor, I was actually challenging the hierarchy (and singularity – another concept to be discussed shortly) of "the right way to know and do mathematics." I did not, however, dismiss those ways from before, but refused to let them dominate my thinking in mathematics. Instead, I engaged in learning and understanding that tended to flatten the hierarchy, and to make the relationship (another concept to be discussed later) between the ways of knowing and understanding mathematical knowledge central to what I valued. From the Traditional Western worldview perspective, this change in valuing of knowledge and ways of knowing would be deemed useless (as the right way is the only and best way); however, from the perspective of an

Indigenous worldview, the leveling of the hierarchy of knowledge would be seen as significant and of great value.

With my friend's attempt to help me with tensor products, the difference in my relationship with knowledge hierarchies and my friend's are made quite clear. Where, as mentioned above, I was seeking to expand what kinds of knowledge I held as worth knowing, and thereby challenging the hierarchy that I had before upheld, my friend was "stuck" within the valuing of that very same hierarchy, unaware, at that point, that it was possible to seek other kinds of knowledge and ways of knowing (for example, in terms of the tensor product), let alone that the alternatives may prove just as valuable, even complementary to his knowledge, rather than subordinate to it. While my seeking of knowledge was changing what so far the concept of hierarchy has represented (and thus its relationship to the Traditional Western worldview and an Indigenous worldview), my friend's response confirms what had emerged about this concept in my grades 1 to 12 experiences.

Likewise, my applications to the education programs of the four different universities, also relate strongly to the hierarchy of knowledge. At the two universities that accepted my applications immediately, there was (a need driven?) hierarchy of knowledge that placed my having a mathematics major above all other knowledge considerations. At the other two universities, there appears to have been different hierarchies of knowledge at play. In the one case this hierarchy placed varying levels of value on different combinations of majors and minors, while at the other university, this hierarchy was more directly related to a hierarchy of completeness of an area of knowledge (in this case, English language courses). These applications' hierarchical valuing kinds of knowledge again reflects what was found in my grades 1 to 12 experiences, with little variation, and echoes how this concept was seen at that time from the perspectives of the Traditional Western worldview and an Indigenous worldview.

Of course, this discussion of the role of hierarchies of knowledge within this part of my story is also tied to the previously noted concepts of specialization, singularity. Within the hierarchical flux that is occurring, my valuing of power and authority, abstraction, categorization and isolation, specialization, relationship, and singularity, as well as the valuing by others, is also in flux. As a consequence of my challenging of the establishment of hierarchies of knowledge, I am also challenging the authority (and power) of the knowledge in question and the push to specialize that knowledge into singular (dichotomized) categories of "right" or "wrong".

Moreover, my challenges to the hierarchy are also challenges to the valuing of abstract, categorized, and isolated knowledge. In this part of my story I am trying to move valued knowledge from the abstract and separated world of the hierarchy to what could be referred to that of a "level playing field". However, I am not just challenging the hierarchical steps to make a flattened field of opportunity, but I am also seeking relationships between myself and mathematical knowledge in different ways. These changes in my stance towards knowledge and ways of knowing is challenging many of the foundations of the Traditional Western worldview and as such a person grounded within such a worldview would counsel me to focus on the singularity of what is most important (in their perspective). These same changes would be openly accepted within an Indigenous worldview as steps towards the gaining and creation of more valuable knowledge.

Thus, from the perspective of axial coding within grounded theory, I have found evidence of not only a merging of the concepts of hierarchy and specialization, but of the merging of these two codes with those of power and authority, abstraction, categorization and isolation, relationship, and singularity. The merger is based upon a very different response to these concepts; however, as, instead of valuing these concepts, I am now challenging the dominant values that have been afforded to them in my life.

Finally, I have coded a new concept for this section, that of context. Context has shown up previously, such as in the context (story) in which my father influenced how I viewed "doing" certain mathematics, but in this part of my story this concept is much more prominent and plays a leading role. My vector calculus experience directs me towards looking for contexts that can lead to greater and different kinds of understanding (such as between the tilt of the heel of my palm of my hand and the slicing of my pencil with my student card). Up until this time, I had not valued context or story, and unfortunately my friend still did not. Context is also central to the knowledge and ways of knowing that were being valued by the universities I applied to. I did not always know the context, but clearly context was defining why there was a difference in what the universities were looking for; there was not a single "right" kind of knowledge or application that defined a successful admission across the four universities, so context must matter. From the Traditional Western worldview perspective, this focus on context as being part of knowledge of value would be seen as distracting and trivial, not worth time considering. However, from an Indigenous worldview perspective, this seeking and valuing of knowledge within context is significant and important. Thus, in the Traditional Western worldview, context is distracting, but from an Indigenous worldview it is enlightening.

One can likely see that context, despite being a new concept in this part of my story, is also inextricably linked to the merging of all the other codes previously discussed. The introduction of context changes hierarchy, it questions power and authority, it emphasizes relationships, it reverses abstraction and categorization and isolation, and it limits the absolute nature of specialization. In this way, all of the concepts coded and remarked upon in this part of the data analysis are connecting together, providing a less disjointed perspective on what can be uncovered in my story.

**"After eight months of being a substitute teacher and volunteer" Analysis**

The next section of my story focuses on experiences I had while teaching mathematics in two different rural schools. Of particular note are those epiphanies I had in relation to the retaining of First Nations students at school and in mathematics classes, to Alge-tiles®, and to the changing of the mathematics course titles in order to gain post-secondary acceptance.

**Prominent features within the data.**

One of the prominent features within this part of my data has also come to be a dominant influence and focus in my life – how First Nations students are viewed within schools and how they view themselves in schools, particularly in relation to mathematics. First, there was the "miscommunication" to the staff at my first school regarding why it was important to keep the First Nations students at our school until October 1st. I choose to call it a miscommunication because I would like to believe that there was some reason other than the appropriation of money that drove these efforts. To this day, even thinking about this incident leaves me feeling noxious, and without further evidence in any direction I will leave the discussion of this incident. I do have grave concerns regarding how schools (and those within them) view First Nations students; however, these concerns are beyond the scope of this dissertation.

The isolation of First Nations students from mathematics is, however, very significant in the particular research that I am presenting right now. Wanting to understand why my First Nations students were dropping out of mathematics (and at times school) prior to grade 12, is one of the main influences on my coming to wonder about the kinds of knowledge and ways of knowing that are valued within mathematics and the teaching and learning of mathematics. These experiences, combined with my enlightenment with respect to the two worldviews that now form the theoretical framework of my research, have been the primary causes for this research I am seeking to make sense of the isolation of so many First Nations students from mathematics, and from there determine how this relationship might be changed.

The next epiphany from this section of my story is my experience with using Alge-tiles® with the one student. Although I had previously started accepting other ways of knowing in mathematics, this was the first time that I had seen such a drastic improvement in the understanding of mathematics when someone else was looking for and using a different way of knowing from the rational approaches most often used in mathematics teaching and learning. This experience strengthened my resolve in the valuing of different kinds of knowledge and ways of knowing in mathematics.

Finally, the unwillingness of post-secondary institutions to consider using a grade 11 credit in mathematics for any of its entrance requirements (in my province), followed by the immediate acceptance of the same course with a grade 12 name was a major epiphany for me. It spoke about the prestige and powers afforded to entrance requirements and to mathematics, and the naïve and insignificant ways that such decisions are sometimes made. It made me question what other potential barriers might exist for students (such as being able to enter specific programs of study) that came down to mathematics.

**Dialogue with the Traditional Western worldview.**

Considering the motive behind the call to keep the First Nations students in school until October 1st came from is difficult from the perspective of either worldview simply because the motive behind it is not an absolute given. Assuming a neutral motive, however, from the perspective of the Traditional Western worldview on calling for such actions within the school would be deemed as reasonable because, regardless of what was being sought in achieving the goal, that goal was presented as the right thing to do. Although no statistical data was provided with the call, it was worded in such a way to give the impression that this goal was grounded in some form of scientific knowledge, making it knowledge of value. Once October 1st was reached, there would be no need to speculate beyond that date since the goal had been reached.

Whereas the goal of keeping the First Nations students attending school until October 1st aligns well with the Traditional Western worldview, the dropping out of mathematics classes by grade 12 by the First Nations students in my school does not. Mathematics as the students saw it, as a logic-based, rational subject, would be seen within the Traditional Western worldview as knowledge (and a way of knowing) of great value, so failing to continue to take mathematics would be seen as a detriment to one's gaining knowledge of great worth. Further, the sentiment of my student who said he now understood "what mathematics wanted from him" would be viewed within this worldview as nonsense. Mathematical knowledge of value is not about relationships or about emotional or even spiritual bonds, it is strictly about logic and rationality. To seek anything else within the gaining of mathematical knowledge would be seen as frivolous and a waste of time.

The third epiphany's relationship to the Traditional Western worldview is a contentious one. From the perspective of this worldview, my introduction of the student to Alge-tiles® and her subsequent use of them over a period of time, would be tantamount to heresy. From the Traditional Western worldview perspective, there is no need for such gimmicks, since the algebra the student was learning has very clear rules that should be learned and applied. Failure to do so was failure to achieve algebraic competence. In the end, this student did come to algebraic competence (and confidence), but she did so by deriving and negotiating the rules that she applied from her interactions with the manipulatives and not through direct memorization and application. Such learning strategies would be seen as challenging the authority of mathematics and of the teacher, while also questioning the notion of the one "right way" to do mathematics. Ultimately, the Alge-tiles® would be seen as removing the abstract nature of algebraic manipulation by suggesting a context for it. Overall, these (and other) manipulatives would be seen by someone grounded within the Traditional Western worldview as unnecessary and even counterproductive.

Finally, I consider the renaming (and consequently the immediate acceptance) of the Math B20 course to Math A30, a purely cosmetic response to the post-secondary refusal to accept Math B20 as an entrance requirement for any program. From the perspective of the Traditional Western worldview, this action and its consequences would not be seen as anything but making sure that the name was right. From this viewpoint, what matters is how things look from the outside, not what is on the inside. Of course, this is very much in alignment with the Traditional Western worldview – the name is external to the knower, and as such it must present the object (the curriculum) with the correct appearance to anyone who hears, sees or reads it. Moreover, the insistence on a grade 12 level an entrance requirement course at a university sends the message of the importance in perceived hierarchies of knowledge.

Overall, the Traditional Western worldview would have mixed responses to the epiphanies within this part of my story. The October 1st deadline would be accepted unquestionably, the First Nations students' dropping out of mathematics (and school) prior to grade 12, and the renaming of the grade 11 course to a grade 12 course title are rational consequences of rational thought and the valuing of the right kinds of knowledge and ways of knowing. The use of the Alge-tiles® by both my student in her learning and I in my teaching, however, challenges the Traditional Western worldview's valuing of the right way of knowing in an abstract and singular way.

**Dialogue with an Indigenous worldview.**

Unlike within the Traditional Western worldview, the goal of keeping the First Nations students in our school until October 1st (still assuming a neutral motive), would likely have been questioned by someone grounded in an Indigenous worldview as they would be wanting to know the context for coming to the particular date. As well, since knowledge is valued for the relationships that it creates, contributes to, and strengthens, a plan for moving forward, for what to be focusing on after October 1st in relation to the First Nations students would be expected; after all, the context of their being in the school does not end at that time. From the perspective of an Indigenous worldview, the singularity and abstraction of one specific date would seem too simplistic for describing what is undeniably a complex set of relationships.

Alternatively to the October 1st goal, the choice by so many of my First Nations students to not take grade 12 mark could quite easily be acceptable within an Indigenous worldview. For these students, as evidenced by the one's comment that he now knew "what mathematics wanted from him," learning mathematics did not connect with them, their lives, or their ways of knowing. Instead, what these students experienced in mathematics classrooms denied almost everything that an Indigenous worldview sought, such as a valuing of diverse ways of knowing and kinds of knowledge, context, and relationships through and for knowledge. For these reasons then, the mathematics that the students were being taught was not a kind of knowledge or way of knowing that they would have seen as of value.

At this point it is probably prudent to return to one of the caveats given earlier in the discussion of the naming of worldviews and membership in them. Although my story speaks to the First Nations students I had, there were other, non-First Nations students who also expressed similar lack of meaning in mathematics for them. Many of these students continued on into grade 12 mathematics classes (mostly because of parental expectations), some passing and some not, but I knew their "hearts" were not in it. I mention these other students to draw to the attention of the reader how the kinds of knowledge and ways of knowing valued by an individual is not dependent upon one's culture or ethnicity, even though the two worldviews in use in my research (and many other worldviews beyond these two) are named after particular cultures (e.g., Western and Indigenous).

My experience using the Alge-tiles® to help my student understand and succeed at algebra, as an alternate way of knowing and kind of knowledge, would be readily accepted within an Indigenous worldview. Not only did the Alge-tiles® provide this student with more diverse ways of knowing, they gave her a way to relate image and concrete representation with the abstract, providing context to the opaque questions that her mathematics classes were filled with.

Finally, the necessity of the renaming of Math B20 to Math A30 would not be valued within an Indigenous worldview. The only relationship that is served by this change is one of perceived authority and hierarchy within entrance requirements. The content of the curricula, which did not change through the renaming of the course, is the story of the knowledge and ways of knowing that are to be valued, not the name. Thus, for a person grounded within an Indigenous worldview, the course renaming would be seen as a trivial undertaking that contributed little, if anything, to the value of the knowledge or ways of knowing.

Again then, it can be seen that viewing my teaching and curriculum piloting years through the lens of an Indigenous worldview gives a reverse correlation to that when viewed through the lens of the Traditional Western worldview. Except for the use of the Alge-tiles®, the feature characteristics of the epiphanies from this part of my story do not align strongly, or at all for the most part, with an Indigenous worldview. However, in my telling of this part of the story, there is strong evidence that my valuing of what is or is not being valued was aligning more strongly with an Indigenous worldview than the Traditional Western worldview.

**Coding and explanation.**

During this part of my story, that of my time as a high school mathematics teacher and as a pilot teacher for the new mathematics curricula at the high school level, most of the same concepts appeared as have been seen in the previous sections. In particular, the concepts of hierarchies, singularity, specialization, categorization and isolation, relationships, power and authority, abstraction, and context and story are all strongly tied to the epiphanies I have highlighted.

The hierarchy of knowledge appears most clearly in the renaming of the mathematics course. By officially changing the course name from Math B20 (indicating a grade 11 course) to Math A30 (indicating a grade 12 course), concerns from post-secondary institutions about whether the course was appropriate for entrance requirements into their programs disappeared. However, there was no content change, just a name change, and so there is an inherent hierarchy within the grade level of courses by (at least) the post-secondary institutions. Knowledge which is not valued when labeled at a grade 11 level becomes valuable in the eyes of the post-secondary institutions when labeled as grade 12. This valuing would seem to be associated with beliefs about perceptions based upon grade level, rather than content of a course.

There are also implicit implications of hierarchies of ways of knowing within the other epiphanies of this part of my story. For example, there is the hierarchy that is present in the First Nations student's response to mathematics, and in particular to where he felt he was within the hierarchy of the kinds of knowledge that are expected and valued in mathematics (rational and logic-based). During this part of my story I also am challenging the notion of these hierarchies of knowledge and ways of knowing through my use of the Alge-tiles® in helping my student come to understand and have success in algebra. At this time I deny the authority of the hierarchy of the right way of knowing and doing mathematics by placing this alternative way on the same level as the abstract and relationship-free approach that the student had originally (by me, at least in part) been taught.

Changing focus from hierarchy, the concept of singularity is evident in the October 1st deadline for keeping the First Nations students in the school. Regardless of the motive behind the setting of the date, there is one single, particular, and specific date that is to be reached. Beyond that date, there is nothing specified, and before it, all that is important is reaching it as described. Further, singularity comes into question in my use of the Alge-tiles® to help teach the one student algebra. By introducing and recommending the use of these manipulatives, I am directly contradicting how I had taught this student previously – that there is one right way to work in algebra and that is to know (i.e., memorize) and follow the rules of algebra. Even the renaming of Math B20 to Math A30 emphasizes a singularity based upon a singular, right name, for the content. It is a singular correctness that stands removed from the needs of the students or the impacts of the decision upon students. The decision meets the need of the post-secondary institutions (the top of the hierarchy in this case), and that is what matters most. In many ways, this also indicates the role of specialization in the hierarchy of knowledge, as the post-secondary programs are designed to take students into more specialized fields of knowledge and their accompanying ways of knowing and the people with that knowledge.

Categorization and isolation is also seen again in this part of my story in a number of ways. First, the deadline of October 1st is specifically labeled and categorized in relation to First Nations students. This marker, nor any other date, is not given for non-First Nations students, and it is associated with all First Nations students, whether from a reserve or not. The First Nations students are in this way segregated from other students, differentiated by their ethnicity and by the relationship to being in school.

This same concept of categorization and isolation is also supported (by the Traditional Western worldview) and questioned (by an Indigenous worldview) in this part of my story through the separation of perceptions about mathematics courses based upon their title. At least for the post-secondary students, but I would also argue for most students, teachers, parents, and the public in general, the title of a course (both grade and identifying name) serves to classify not only a particular kind of mathematics, but a hierarchical level of perceived difficulty.

Relationship also plays a major role as a concept in this part of my story. In the case of the Alge-tiles® use, relationships between the student and mathematics is emphasized, while in the case of the renaming of the course, the possibility of there being a relationship between the name given to a course and its grade level is downplayed, perhaps even dismissed.

The concept of relationship is of greatest importance; however, in the epiphany I shared regarding the avoidance of mathematics by my First Nations students, particularly at the grade 12 level. For at least one of my students, this retreat from mathematics classes and learning is all about relationship. In fact, it is about failed relationships between the students and what they have experienced the purpose and way of being of mathematics to be, that is, no relationship at all. In at least his learning of mathematics, the particular student had been frustrated throughout all of his mathematics classes trying to understand what kind of relationship he was to have with mathematics ("what mathematics wants from me"). Ultimately, he realized that therein lay the inherent problem – mathematics did not want a relationship with him. It wanted to remain isolated, yet known by him, in an abstract and decontextualized way.

Power and authority also runs through this part of my story as a major concept. The power and authority of the singular date of October 1st determined how teachers were to interact with their First Nations students, and what their goal in those interactions was to be. Similarly, the lack of relationship between the one First Nations student and mathematics, as he had come to know it, was also based upon power and authority. His comments to me indicated that despite having come to understand what was expected from him, under the guise of the authority of mathematics and how it was taught to him, he did not have the desire to meet the demands of that authority. Thus, by not taking mathematics classes beyond grade 11, this student was both giving in to the power and authority of the mathematics by conceding “victory” to it, and challenging that same authority by refusing to give in to that authority by relinquishing the kinds of knowledge and ways of knowing that he valued. Finally, the power and authority of the valuing of specific kinds of knowledge and ways of knowing within the teaching and learning of mathematics was challenged in my introduction of the one student to Alge-tiles® and in her refusal to give in to “the right way” of knowing and doing algebra.

The last two concepts present in this portion of my story are abstraction and context. As can be noted throughout the prior discussion of the coding for this section, the notions of abstraction and context frequently entered into the discussion of the other concepts. However, these two concepts are even more strongly present in how they give rise to each other throughout the various epiphanies. For example, it is in his seeking of the context through which he can relate to mathematics that my one First Nations student came to understand that mathematics, as he was being taught and learning it, denied context in order to create knowledge that was considered of more value because of the high regard given to abstract knowledge within his classes.

Conversely, my other student’s use of Alge-tiles® to learn to do and understand algebra places context in the teaching and learning of mathematics at most on a level par with abstract knowledge. In fact, this epiphany stresses the relationship between contextual and abstract understanding, as the student gradually abstracted her understanding, putting the Alge-tiles® away after developing her confidence in the rules and understandings she had created using them.

Even in the renaming of the new mathematics course from Math B20 to Math A30 context and abstraction play a role.  Context is what gave rise to the original naming of the course; that is, the students who took this course would mostly be in grade 11 and the bulk of the course material came from previous grade 11 curricula documents (specifically, Algebra 20 and Geo-Trig 20).  Abstraction, however, called for the changing of this name.  From an abstract perspective it had been decided that a grade 11 credit, regardless of the mathematical content, was not appropriate for the entrance requirements to a post-secondary program of study.  Simply put, mathematics courses serving as entrance requirements to post-secondary institutions must be grade 12 credits.

Overall, the relationships and connections between the two worldviews (the Traditional Western worldview and an Indigenous worldview) continue to have the same alignment to each of these concepts as seen before.  There is one notable exception in considering the concepts from the worldview perspectives, and that more than ever before, my epiphanies dramatize perspectives aligned with an Indigenous worldview that are directly challenging those that align more with the Traditional Western worldview.

Further, the discussion of the coding explanations also emphasizes that the various codes revealed thus far are continuing to interact with and impact each other.  None of the codes at any time stand alone; rather, they work together in various ways to capture the concepts depicted in my story.  Thus, the axial coding noted previously related to the merging of the codes into a broad category continues on in my story of teaching and piloting mathematics in the two rural schools.

**"My experiences as a pilot teacher and implementation leader" Analysis**

In the next section of my story I shared memories of experiences I had during the time that I worked as the K to 12 Mathematics Consultant at the Ministry of Education while also working towards my Master's degree.  It is during this time I had the greatest number of epiphanies of direct consequence to my current research.

**Prominent features within the data.**

My auto/ethnographic analysis of this time period in my story lead to the identification of the first epiphany I will speak to – that of my encounter with how my colleagues at the WNCP CCF renewal table responded to my suggestion to include zero in the earliest grades of curriculum outcomes.  I was completely caught off guard by the opposition to having students learn about zero in the early grades, as it seemed to me to be a concept that they could not only handle (from the perspective of representing a quantity of none) but, that it would be beneficial in developing the students' understanding of the base-ten place value system that they would be working with for the rest of their lives.

My curiosity regarding the heated opposition to my proposal to include zero in the earliest years of the mathematics curricula ultimately lead me to find out about the history of zero. I was amazed to read about the variety of ways that different cultures, and different individuals within the cultures, responded to and dealt with zero. As I had seen earlier, both in elementary school and during my pursuit of a mathematics degree, the history of zero again brought together mathematics and religion. At times, this encounter was positive (such as with India's whole-hearted embracing of zero when it arrived as a place holder from Babylonia) and at other times it resulted in fear and disguise (such as when the Hindu-Arabic number system first reached Greece and where it was both rejected and used clandestinely). In both cases, the acceptance or rejection of zero was driven by religious beliefs. In Greece, these religious beliefs were themselves tied to philosophy and mathematics, making the divide between religion and zero even greater.

In addition, a vast majority of the epiphanies that I have noted within this section of my story again link to situations, questions and issues related to First Nations and Métis students and cultural relations to mathematics. The first of these epiphanies is my digging deeper into the dropout rates of First Nations students with respect to mathematics (and school, in general). It was during this time that I started to see the larger picture of the divide between these students and mathematics was not only common to the first school I taught in, but was actually an "epidemic" across the province, Canada, and the world (in relation to Indigenous students). Whereas much research into mathematics was continuously revealing that all humans are mathematical beings (Butterworth, 1999; Devlin, 2000; Wynn, 1992), the data I was finding seemed to deny this inherent human relationship to mathematics to Indigenous peoples, and this was a conclusion I was not willing to accept. In many ways the apparent divide between "all humans are mathematical beings" and the struggles of so many Indigenous students with learning mathematics was too reminiscent of Smith's (1999) comments on how humanism was often used to "Other" certain groups of people, presenting them as sub-human, or less valuable beings, and I refuse to accept such a classification of Indigenous peoples, or to anyone.

Connecting to the struggles of the Indigenous students with mathematics (as well as likely to other students who struggle in mathematics) is my epiphany with respect to First Nations and Métis ways of knowing. As epiphanies go, this particular one (at this time) was disappointing because I could not find out, let alone understand, what was meant by First Nations and Métis ways of knowing. It was a concept that I was eager to understand, but also one that continued to allude me.

My work with the First Nations and Métis teachers and elders during the mathematics curriculum renewal provides evidence regarding my attempts to understand what these First Nations and Métis ways of knowing were and how they might fit into the teaching and learning of mathematics. Each of the failures and successes (partial or otherwise) that emerged through our work together steadily helped to introduce me to aspects of First Nations and Métis ways of knowing. For example, I learned that such ways of knowing value story and context, and that without either one, the knowledge sought or used in mathematics was isolated from the student and important relationships were thus not forged. I also learned that context and relationship to connections were important within First Nations and Métis ways of knowing. Moreover, I realized that these alternative ways of knowing also related to how mathematics was represented and applied.

Also related to the renewal of the mathematics curricula in Saskatchewan, is the epiphany resulting from the reflections of Dr. Edward Doolittle while we were discussing the draft of one of the curricula documents. Dr. Doolittle's revelation to me that the English spoken in many First Nations and Métis communities did not necessarily carry the same meanings for words as when I speak them was astonishing to me. Despite my knowledge that translation between languages is rarely, if ever, a direct one-to-one correspondence, I had never considered that the result of translation might be the assigning of different meanings to words in the new language. However, this is not to say that it does not make sense that this would happen, or to imply that it is a negative consequence of translation. In fact, especially in cases of forced language denial, as the First Nations and Métis of Canada experienced starting with the first days of colonization, I would argue that the polysemy that Ed had brought to my attention is a powerful and important consequence. It is within the resulting polysemy that much of the original First Nations and Métis languages might still exist, continuing to deny the oppressive authority of colonization and its processes.

All of these previous epiphanies relating to First Nations and Métis students struggles with mathematics and ways of knowing and doing mathematics are brought together into one story, that of Louise Poirier and her sharing of her work with the Inuit of northern Quebec with me. This particular epiphany is full of examples of First Nations and Métis kinds of knowledge and ways of knowing. At the surface, there is the Inuit base 20 (sub-base 5) number system, which is a completely different way of knowing and representing quantity from what is traditionally taught in Western schools. Going deeper into this different way of thinking about and knowing quantities, however, there exists other aspects of quantity within the language of the Inuit of northern Quebec that do not exist in the English language nor the base 10 number system. The recognition of context within the identification and naming of quantity is a feature of the Inuit number system that is not present in the number system taught in most schools around the world. Further, the doing of mathematics orally only, with no symbolic representation system existing, also challenges what is typically taught in schools about how to do mathematics. Thus, Louise's story opened my eyes to mathematical possibilities that I had never before considered.

My second last epiphany from this section of my story moves from a focus on First Nations and Métis students and mathematics to the broader perspective of ethnomathematics, or mathematics that is situated and grounded within culture. As such, the previous discussions belong to ethnomathematics. For me, this was a welcome set of knowledge to be introduced to, as it provided a home for the many changes in my thinking and doing of mathematics that had occurred up to this point. Within ethnomathematics, I found justification for my valuing of different kinds of knowledge and ways of knowing in mathematics, as well as gaining valuable insights taking me further into pursuing these possibilities.

Finally, my growing awareness and reflection upon how mathematics has become a gatekeeper for many pursuits in today's world, is another crucial feature from this part of my story. Through requests that I received to update and correlate job descriptions and program entrance requirements so that decisions could be made about applications coming from individuals who had gone through a different mathematics program, the level of authority in defining a successful applicant that was given to mathematics was shocking. For example, in the case of the administrative position looking after the people doing tree maintenance in an urban setting, the level of mathematics required was not based on the mathematics they needed to understand and be able to do; rather, the mathematics was being used to position the applicant in a hierarchy of proficiency somehow, but not obviously, deemed to be tied to the level of mathematics achieved. Similarly, this same kind of hierarchy seems to have been in place in the mathematics requirement for a stenography course. Stenography, as a profession, would not seem to be reliant upon mathematics (particularly not high school or university level mathematics), yet applicants to a stenography program were required to have credits for specific abstract and theoretical mathematics courses. This requirement appears to have been solely for the purpose of gauging some kind of intelligence, and perhaps perseverance, that an applicant had; an intelligence that does not require doing mathematics, but does require being able to do mathematics, and only mathematics. When I consider these barriers that were being created by the seemingly unsubstantiated requirement for particular mathematics courses along side the struggles of First Nations and Métis students with mathematics, my concerns about both situations are intensified.

Having completed my highlighting of the pertinent features of the epiphanies of my story during my time at the Ministry of Education and while working on my Master's degree, I can now turn to the hermeneutic analysis of this part of my story and this discussion. I begin by considering how these epiphanies relate (or do not relate) to the foundations of the Traditional Western worldview.

**Dialogue with the Traditional Western worldview.**

A Gadamerian hermeneutic analysis of this part of my story in relation to the Traditional Western worldview reveals both connections and divergences. My WNCP colleagues' rejection of including zero in the earliest grades of the mathematics curricula correlates well with the foundations of the Traditional Western worldview, especially in light of there being a hierarchical (chronological) order in which mathematical ideas were developed, and that particular order must necessarily be the right order for students to be taught mathematics as well. In addition, an assumed hierarchy between cultures can also be seen to be at play either within my colleagues thinking or within what is accepted as general knowledge about the development of zero. The late arrival of zero into the number system was only true in the Greek world (and those who followed it's knowledge). The Mayan's had used zero centuries before it was even considered (and rejected) within Greek society. Thus, from the perspective of the Traditional Worldview, the belief that zero developed late historically is based upon a hierarchical view of the value of knowledge between different cultures. From the perspective of the Traditional

Western worldview, the early incorporation of zero into Mayan mathematics and even Indian mathematics holds no significance (and hence, no real value) in comparison to when zero was accepted into Greek mathematics.

Likewise, the claim that zero was too difficult of a concept for elementary students is also strongly situated within the hierarchical view of knowledge within the Traditional Western worldview. Through such a view, mathematical knowledge is seen as falling into a hierarchy of difficulty (complexness) based upon both the ideas present and where within history the concept was developed.

When actually looking into the historical reasons for the immediate or delayed acceptance of the concept of zero as both a place holder and a quantity does not likewise connect with the perspectives of the Traditional Western worldview. By relating the valuing of knowledge about zero to religion (by the Greeks and the Indians) as the basis of deciding whether it should be incorporated into mathematical knowledge, in general, is based on the valuing of religious (and spiritual knowledge) which the Traditional Western worldview explicitly distances itself from. Knowledge which is not rational or logic-based and which cannot be substantiated through physical representation and measurability, which is how religious knowledge is viewed from the perspective of the Traditional Western worldview, is not knowledge of value.

Alternatively, from the perspective of the Traditional Western worldview, the high dropout rates of First Nations students in relation to mathematics education would be seen as a clear indication of a failure, on behalf of the students, to do mathematics “the right way”. Since mathematical knowledge is both rational and logical in origin, the problem would necessarily be placed squarely upon the ability and commitment of the students only.

Further, the notion of singularity of the “right way” to know and do mathematics within the Traditional Western worldview would contradict my interest in, and pursuit of, understanding First Nations and Métis ways of knowing. The arguments given by my Ministry colleagues was that these ways of knowing were different from the rational and logical ways of knowing that are valued within the Traditional Western worldview, so from that worldview perspective, what these other ways of knowing were was of no consequence; these alternative ways of knowing would necessarily be of no real value. Thus, a person grounded within the Traditional Western worldview would outright dismiss consideration of the First Nations and Métis ways of knowing that I was trying to understand.

The vetting sessions with the First Nations and Métis teachers and elders during the renewal of the k to 9 mathematics curricula would also not be valued from the perspective of the Traditional Western worldview. The purpose of these sessions was to try to capture within the curricula First Nations and Métis ways of knowing as well as contexts and perspectives relevant to First Nations and Métis students. Within the Traditional Western worldview, such extra knowledge and grounding of knowledge distracts from the actual knowledge and ways of knowing – the rational and the logical. Everything else would be viewed as extraneous and pointless.

Likewise, the Traditional Western worldview would not value the polysemic meanings used by First Nations and Métis when speaking the English language. In fact, such alternative meanings would be viewed as not just unnecessary, but also as challenges to the hierarchical positioning of the English language and the authority that it thus holds. The polysemic meanings to words such as equal would be shunned and eliminated (whenever possible) by a person grounded within the Traditional Western worldview. In the particular case of the world "equal" the negative response to the polysemy by the Traditional Western worldview would be immediate, since in the "right way" of knowing, the polysemic meaning being given in to equal in First Nations and Métis communities belongs to the word "equity". Two words need not be given the same meaning when the "true" meanings of each word suffices in all instances of use.

As the Traditional Western worldview is grounded in the valuing of the singularity and abstraction of knowledge, as well as their being "one right way" to do mathematics, Louise's story of the number system and mathematics of the Inuit of northern Quebec would be viewed as possibly interesting, but overall not contributing to mathematical knowledge. The inclusion of context within their working with numbers would make the mathematical knowledge of the Inuit of northern Quebec as too diffused and not abstract enough to be of real value. Further, by positioning the doing and knowing of mathematics in strictly oral circumstances would also be seen as a deficiency from the perspective of the Traditional Western worldview. Overall, the mathematics of the Inuit of northern Quebec that Louise shared with me would not be seen as having any real value by a person grounded within the Traditional Western worldview, and if working in Louise's situation, such a person would have sought ways to change Inuit knowledge so that they knew and did mathematics using the correct and most valuable base 10 number system.

With the move from this particular Inuit mathematics knowledge and ways of knowing to the broader landscape of ethnomathematics, the previously discussed divide between what I was experiencing and what is valued by the Traditional Western worldview continued to grow. My initial foray into ethnomathematics took me deeper into consideration of different kinds of knowledge and ways of knowing that can be valued in mathematics, and thus moved me further from the Traditional Western worldview which would view such dalliances as a waste of time. From this worldview perspective, there is no need to explore other possibilities when the "right way" (rational and logical), was already known and given to me.

Finally, the requirement of needing to have mathematics beyond the mathematics that will be used within particular jobs or programs of study would likely not raise an eyebrow in concern for people grounded within the Traditional Western worldview. From their perspective, it would seem obvious that such a requirement should be in place because the mathematical knowledge that is required would position the knower high up in the hierarchy of learning and knowledge. Thus, applicants with the required mathematics courses would be seen to be competent learners, capable of learning the "right way" to do things and to be proficient in doing them.

As has been noted in the previous analyses of different sections of my story, the Traditional Western worldview both aligns and contradicts with the experiences I had during my work at the Ministry of Education and a student pursuing their Master's degree. Further, my positioning in relation to these experiences also puts me in different relationships with the Traditional Western worldview. Next, I will consider how the epiphanies within this section of my story compare to the values of an Indigenous worldview.

**Dialogue with an Indigenous worldview.**

Considering the epiphanies from this part of my story through the lens of an Indigenous worldview, a much different set of relationships between the story and worldview emerges from those relating the same epiphanies and the Traditional Western worldview. In the case of my WNCP colleagues response to the suggestion to include zero as part of the mathematics outcomes in the earliest grades, a person grounded within an Indigenous worldview would question the temporal authority assigned to any knowledge. That is to say, within an Indigenous worldview, the relevance of the knowledge to the person, their place and context (hence their time, not the time of others) determines the value of knowledge. Within an Indigenous worldview, a person's ability to use and value any knowledge is dependent upon who they are and where they are at, and not on the experiences of others.

From the perspective of an Indigenous worldview, the categorizing of mathematical knowledge into hierarchies of complexity would also be seen as an unnecessary and limiting attitude towards knowledge of value. One's current context and needs defines the value of particular knowledge. How complex or simple that knowledge is does not influence the value assigned to the knowledge at a particular time since if it is knowledge that is needed then it is knowledge of value. Thus, a person grounded within an Indigenous worldview might argue that since students in the early elementary years need to use zero in the writing of whole numbers, and since they have experiences in their daily lives that involve their recognition of having none of something, zero is a concept that is appropriate for these children to learn.

Likewise, the relationship between religion (and possibly spirituality) and zero would not devalue knowledge about zero within an Indigenous worldview. Instead, religious reasons either supporting or questioning the inclusion of zero within mathematical knowledge would be viewed as valuable contributions to the diversity of knowledge and understanding being constructed around zero. It should be noted, however, that the exclusion of zero, solely on the basis of religious reasons would likely be viewed as a limiting stance regarding the perception of value for knowledge about zero. Just as an Indigenous worldview would accept, but also question knowledge that is confined to a different way of knowing, such as logic, so too would it accept but also question the devaluing of knowledge on the basis of only a single way of knowing, in this case religion.

Furthermore, an Indigenous worldview would dismiss the historical arguments in relation to when knowledge about zero is appropriate for students. Students' contexts make the knowledge of value, not someone else's past experiences. Furthermore, the valuing of one culture's mathematical knowledge over that of another in determining the emergence of any mathematical concept would be seen as limiting from an Indigenous worldview perspective because the diversity of ways of knowing and kinds of knowledge would not be valued equitably.

When considering the high dropout rates for First Nations students through the lens of an Indigenous worldview, the question of what do the students have to say about the situation would arise. There would not be an automatic assumption that there is a deficiency in the students causing their struggles and separation from mathematics, because the assumption that the way mathematics is taught and learned, even what mathematics is or can be, would not be assumed to be singular and unrelated to the learner and knower. Instead, a person grounded within an Indigenous worldview would likely investigate what is being valued and whether there are other ways of mathematical knowing and other kinds of mathematical knowledge that could also be valued.

Likewise, the suggestion by my Ministry colleagues that there are First Nations and Métis ways of knowing would be of great interest to a person grounded within an Indigenous worldview. From that person's perspective, alternative ways of knowing are important to consider, and even embrace, so that the knowledge one creates is the knowledge of most value for your particular place, time, and context. Thus, such a person would encourage my pursuit of understanding of these alternate ways of knowing.

Consequently, the holding of the curriculum renewal feedback sessions with Nations and Métis teachers and elders to try to find ways to infuse First Nations and Métis content, perspectives, and ways of knowing into the mathematics curricula would be well received by a person grounded within an Indigenous worldview. These sessions encouraged the sharing of different kinds of knowledge and ways of knowing, and were an attempt, in reality, to permeate the valuing of alternative knowledges and ways of knowing throughout the curricula documents. From the perspective of an Indigenous worldview, such actions concretize the seeking of knowledge and ways of knowing that are most valuable.

Similarly, an Indigenous worldview would be receptive to the polysemic meanings that Edward referenced in our discussion about the way the word "equal" is used in First Nations and Métis communities. Within an Indigenous worldview, a person's context, which includes how they think and speak about the context, is of great value when seeking knowledge and ways of knowing.

A person grounded within an Indigenous worldview would also be happy to hear of Louise's work in trying to value the mathematical knowledge and ways of knowing of the Inuit of northern Quebec. Such a person would be further pleased to note that Louise's work also focuses on trying to find authentic and respectful ways to have students also learn about mathematics within a base 10 number system and the related representation of such mathematics.

Such a valuing of the kinds of mathematical knowledge and ways of knowing of both Western schooling and of Inuit communities in northern Quebec would be accepted within an Indigenous worldview as commitments to working towards not limiting, but rather seeking, alternative kinds of knowledge and ways of knowing.

My move into exploring the field of ethnomathematics would also be viewed favorably from the perspective of an Indigenous worldview. An Indigenous worldview's seeking and valuing diversity in kinds of knowledge and ways of knowing within any context would approve of and support the diversity of ways of knowing and kinds of knowledge found within ethnomathematical research and discussions.

The same, however, cannot necessarily be said of the mathematics requirements for jobs and programs of study that do not align with the mathematical needs within those areas. A relevant context for needing these particular mathematics courses is not present within the requirements. Moreover, from the perspective of an Indigenous worldview, which values diverse ways of knowing and kinds of knowledge, the assumption that mathematics courses are the only way one can prove their ability to be successful in furthering their position at work or in their schooling would be seen as unfounded. Instead, these requirements would be regarded as unnecessarily adding barriers and limitations upon people seeking to enter into these areas, thereby oppressing individuals on irrelevant (and unsubstantiated) grounds.

From the perspective of an Indigenous worldview, then, this part of my story has also highlighted experiences and values that are both supported and questioned (although never fully rejected). From this part of the analysis, I now move on to using grounded theory methods to code the concepts I found throughout this part of my story and my analysis of them.

**Coding and explanation.**

Once again, consideration of the concept coding of this section of my story reveals the same concepts and the merging of those concepts as have been seen before. Hierarchy, specialization, singularity, categorization and isolation, relationship, power and authority, abstraction, and context/story regularly emerge as concepts underlying the various epiphanies that have just been discussed.

Hierarchy appears in the coding of this section of my story in a number of ways. The job and programs of study mathematics requirements create an artificial hierarchy of applicants by using mathematics courses that have no relevant content in relation to the situation as the determining factors for whether an application is even considered.

A different kind of hierarchy, one based on chronology, is also in play in the WNCP considerations of the order in which mathematics content should be taught. In the case of teaching students about zero in the beginning years of their mathematics learning, the argument is made that this is not appropriate because zero was not developed early in mathematics. Thus, history defines a chronological hierarchy of events, and teaching and learning is meant to follow that same sequence. In this case, another hierarchy lays hidden below (and supporting) the chronological one, namely that of a cultural hierarchy that classifies the relative value of mathematical knowledge in different cultures in relation to each other. This cultural hierarchy is evident in how the chronological hierarchy of mathematics development ignores many cultures' mathematical knowledge and ways of knowing, such as those of the Mayans and the Indians. This section of my story also raises the notion of a hierarchy of mathematical difficulty. This hierarchy is likewise supported by the chronological and cultural hierarchies previously mentioned.

Even languages appear to be placed in a hierarchical relationship, with English having dominance over First Nations and Métis languages. It is this presumed authority of the English language over the other languages that ultimately gives rise to the polysemic meanings for English words used in First Nations and Métis communities that Dr. Doolittle brought to my attention.

The concept of specialization again goes hand-in-hand with the concept of hierarchy within this section of my story. The dominance given to (historical) Greek knowledge and, as a result, Greek chronology, speaks to an assumption of specialization within Greek knowledge and ways of knowing. In particular, the knowledge and ways of knowing associated with Greek developments are considered the most specialized in their abstractness and authority. Similarly, the hierarchy of mathematical difficulty presumes greater specialization, knowledge, and thus authority for any person who is able to progress upwards through the hierarchy.

Much of the discussion and analysis of this section also is grounded within either the pursuit of or, more frequently, the challenging of singularity in knowledge and ways of knowing. My many encounters with the notion of First Nations and Métis ways of knowing are in particular challenging of the notion of knowledge of value being singular in nature (as is held by the Traditional Western worldview). This concept is perhaps most evident in Louise's story of her encounters with aspects of the mathematics of the Inuit of northern Quebec, such as their emphasis on context determining how the mathematics is communicated and done.

Within this part of my story, the concepts of singularity and hierarchy also attach to the concept of categorization and isolation. Parts of the story, as previously noted, categorize mathematical knowledge into hierarchies that isolate different pieces of mathematical knowledge from each other thereby strengthening the perceived singularity of the knowledge pieces. This isolation of knowledge became particularly noticeable during the feedback sessions held with First Nations and Métis teachers and elders. During this time, they expressed extreme concern about the limitation upon the infusing First Nations and Métis content, perspectives, and ways of knowing into the mathematics outcomes and indicators because of the over-riding mandate of the use of single sentences and of limiting details in examples. For those involved in this process, these limitations prohibited connecting of the outcomes together and to students' lives in meaningful ways.

Relationship was also a frequent and strong concept that again merged in this part of my story. First, the statistical data seems to imply a harmful relationship between many First Nations and Métis students and mathematics as they drop out of mathematics, and thus more barriers are put into place for their futures. Further, the feedback sessions for the curricula renewal brought to light how the avoidance of context or story prevented students from finding meaningful ways to build a relationship with mathematics, denying them the possibility to understand "what mathematics wants from [them]." As well, the (possibility of a) relationship between religion and mathematics was again present in this section of my story. Finally, this part of the story marks an even greater distancing of myself in my valuing of knowledge and ways of knowing from the Traditional Western worldview, and the continued strengthening of my relationships with an Indigenous worldview.

As noted in the discussion of the concepts of hierarchy, specialization, and singularity, authority and power also is a major concept within this part of my story. What my analysis has brought to light is the power and authority that has been given (by some) to the English language and (historical) Greek culture, and some of the consequences arising from wielding that supremacy. It also gives some examples in which this same power and authority is being challenged, such as through ethnomathematical research and passively (almost unnoticeably) through the polysemy found in some First Nations and Métis communities when English is spoken. In addition, this part of my story speaks of temporal (chronological) authority that often defines when different knowledge is to be learned.

With hierarchical, singular, and categorized and isolated knowledge being sought and valued, abstraction also becomes a significant concept to consider. In my epiphany related to the barriers that mathematics can create, there is evidence that mathematics at a particular grade, regardless of specific content, has been abstracted to be a reflection of the level of a person's ability to gain and use knowledge in significant ways. The higher the grade of mathematics achieved, the more reliable a student or worker will be. The concept of valuing abstraction of knowledge is also challenged within this part of my story. In particular, Louise's experience with the concrete and contextual mathematical knowledge and ways of knowing of the Inuit students from northern Quebec, and their efficiency and fluency in working with that knowledge and ways of knowing, raises the question of whether the valuing of abstract knowledge over all else is actually substantiated.

Finally, the concept of context (story) is again present throughout this section of my story. It exists in the lack of contextual attention in the historical considerations given to zero, as well as in the occurrence of polysemy within First Nations and Métis communities. Context is also central to the mathematics of the Inuit of northern Quebec that Louise told me about. In some cases, context specifically is excluded and in others it is explicitly included; however, it is frequently a central feature within the epiphanies of my story.

As my analysis of my story continues, it is becoming harder and harder to keep the various concepts isolated from each other as how each concept is encountered within the story is dependent upon and informs what is also present in relation to the other concepts. Thus, axial coding is quickly coming to dominate the grounded theory analysis of my story. I am, however, avoiding the naming of the categories originating within these conceptual interactions and mergers. I choose to not label the categories at this point because I know that even after the next, and final section, of my story is analyzed, the categories will only be representative of the first section of my analysis, that of my story (the auto/ethnographical part of my research). The methods of grounded theory, however, will also be taking me into the analysis of data that comes from other sources, and I do not want to specify any categories until I have had a chance to engage in those analyses as well.

**"In the first term of my PhD program" Analysis**

This final part to my story focuses on my starting into the PhD program for which this dissertation is being written. It is a small section that bridges over into my current research.

**Prominent features within the data.**

Within this part of my story, there are three epiphanies. The first epiphany, that of taking the Trends and Issues in Mathematics Education course, provided me with safe opportunities to bounce my evolving understandings of what is, can be, and even should be valued in terms of the kinds of knowledge and ways of knowing in mathematics and the teaching and learning of mathematics, off of different individuals.

The second epiphany, the Decolonizing Aboriginal Education course, although still a safe place to express feelings and thoughts, was continuously challenging my beliefs, my knowledge, and my ways of knowing, both with respect to the impacts of colonization on First Nations and Métis and to the impacts (including privileges) of colonization upon me. Over and over, this course connected to and reinforced my valuing of alternative ways of knowing and kinds of knowledge, particularly in mathematics (despite my being the only person in the class who ever mentioned it).

It is out of the Decolonizing Aboriginal Education course that the final epiphany of my story emerged, the reading of Leroy Little Bear's (2000) *Jagged Worldviews Colliding.* This reading invited me to consider what kinds of knowledge and ways of knowing are valued within the Traditional Western worldview and within an Indigenous worldview in relation to mathematics and the teaching and learning of mathematics. It also provided me the foundations for a framework through which to consider such values in mathematics and the teaching and learning of mathematics, and ultimately it is this reading that became the foundation for the research I am now engaged in.

**Dialogue with the Traditional Western worldview.**

From the perspective of the Traditional Western worldview, all three of my epiphanies would be evidence of time and effort wasted. A person grounded within this worldview would see no need to study trends or issues in mathematics education, as the "right way" to learn and do mathematics is already known. This right way emphasizes the abstract, compartmentalized, hierarchical, and singular knowledge and ways of knowing that are based in rational thought and logical reasoning.

Likewise, a person grounded within the Traditional Western worldview would not find justification in spending time considering the decolonizing of education; after all, education is being done in the "right way." If anyone was experiencing negative impacts from the education system it would be because they were not valuing the abstract and authoritative knowledge and ways of knowing that the system brings to its student. Moreover, no one is privileged over another if they are valuing the correct knowledge and ways of knowing.

Finally, the Traditional Western worldview would also refuse to assign any significance or importance to the work of Leroy Little Bear. Little Bear, in writing about and contrasting the two worldviews would not be seen as contributing to valuable knowledge, since the Traditional Western knowledge is assumed to be correct (the "right way" of looking at things), while an Indigenous worldview offers nothing of value that is not already in place as a consequence of the Traditional Western worldview.

Thus, my considerations of context, story, and relationships within all three of the epiphanies would appear as frivolous or even sacrilege within the Traditional Western worldview. Great concern would be expressed about the likelihood that these courses and readings would ultimately lead to the questioning of the authority, specialization, and power that are central to mathematics and the teaching and learning of mathematics, which they did. A person grounded within the Traditional worldview would question my need to seek the understanding that I gained in these epiphanies, reminding me that knowledge of value comes from isolating and distancing oneself from what is to be known and not relating oneself to the knowledge. I was not seeking knowledge for the sake of knowledge, and that ultimately calls into question the validity and worth of the knowledge.

**Dialogue with an Indigenous worldview.**

From the perspective of an Indigenous worldview, my focus, investigations, and learnings within the Trends and Issues in Mathematics Education course would be seen as valuable pursuits of knowledge and ways of knowing. Likewise, my new experiences and knowledges emerging from the Decolonizing Aboriginal English class would be seen as strengthening the diversity of my knowledge and ways of knowing, not only in terms of mathematics and the teaching and learning of mathematics, but in relation to life and relationships within life. As a specific context for these learnings, Little Bear's (2000) writing would also be valued as a further extension of my new knowledges, and ways of knowing; it would be appreciated for providing me a new context of understanding and for broadening the diversity of my knowledge.

According to an Indigenous worldview, these three epiphanies would be seen as important because they explicitly challenged any assumptions of absolute power and authority associated with the construction and maintenance of hierarchies, categorization, abstraction, and decontextualizing of mathematical knowledge and ways of knowing. The knowledge I was seeking and gaining in these contexts would be viewed as good knowledge, and knowledge for the sake of the greater good.

**Coding and explanation.**

Thus, although far less detailed then the rest of my story, the three epiphanies from the start of my entering the PhD program again related to the previously identified and described conceptual codes: hierarchies, specialization, singularity, categorization and isolation, relationships, power and authority, abstraction, and context (story). Moreover, how each of the two worldviews would respond to each of these concepts is in the same ways as previously discussed.

Likewise, the interplay and interdependence of each of these concepts also continues to play out in the same ways through these three epiphanies, giving even a stronger basis upon which to claim the emergence of axial coding, the grouping together of initial concept codes into a larger conceptual category. This larger conceptual category is also becoming clearer through the final epiphanies of my story, that of the purpose of seeking knowledge. In particular, this conceptual category focuses the question of how are these concepts related to the kinds of knowledge and ways of knowing of importance? For now, however, I will hold back on further clarifying and naming this broader conceptual category as it appears to be merging; rather, I will wait to see what emerges from my data that comes from non-personal sources, allowing the individual concepts, and this emerging conceptual category, to evolve without my overt interference.

**My Story: Summary of the Analysis and Moving Forward**

Within analyzing and coding data in grounded theory, one is seeking for “saturation” of themes within the data, and in the proceeding analysis of my story, such saturation can be seen in all of the emergent codes: hierarchy, specialization, singularity, categorization and isolation, relationship, power and authority, abstraction, and context (story). Once each of these concepts emerged within my story, they continued to play a role within my analysis of the rest, and the elaborations about the codes also quickly became consistent.

If this research was purely auto/ethnographic, I would now be content to head toward my conclusions, but the question that I am asking, "what kinds of knowledge and ways of knowing are of value in mathematics and the teaching and learning of mathematics" and the implications of the answer to that question, has not been addressed by simply considering my personal story.

This is where grounded theory, beyond conceptual coding, now takes hold of my research by requiring that I now continue on by researching areas that my initial set of data analysis and concepts leads me to. As a slight deviance to the common use of grounded theory, however, my original data set is not leading me to investigate one arising area of data; rather, it has me now wishing to explore four areas outside of my personal context:

1) how people view and think about what mathematics is (the philosophies of mathematics),
2) show it is believed that mathematics should be taught and learned (the math wars),
3) how mathematics relates to culture and individuals (Indigenous students' mathematics struggles and ethnomathematics), and
4) how curriculum represents mathematics content with an eye to emerging mathematics content (risk education).

In so continuing with the documentation of these sets of data, each will be followed by an analysis using the same procedures as was used in the analysis of my story: an identification of the prominent features or epiphanies found within the data, the results of a Gadamerian hermeneutic dialogue between the two worldviews (the Traditional Western worldview and an Indigenous worldview) and the data presented, and a conceptual coding and related descriptions emerging from the epiphanies and dialogues.

Thus, although officially moving away from the auto/ethnography of my collage of methodologies, I still carry forward the notions of epiphanies and prominent features from that methodology to maintain consistency and clarity within the rest of my analysis. With these understandings of how that presentation and analysis of data will now proceed, I move to the next data set – the philosophies of mathematics.

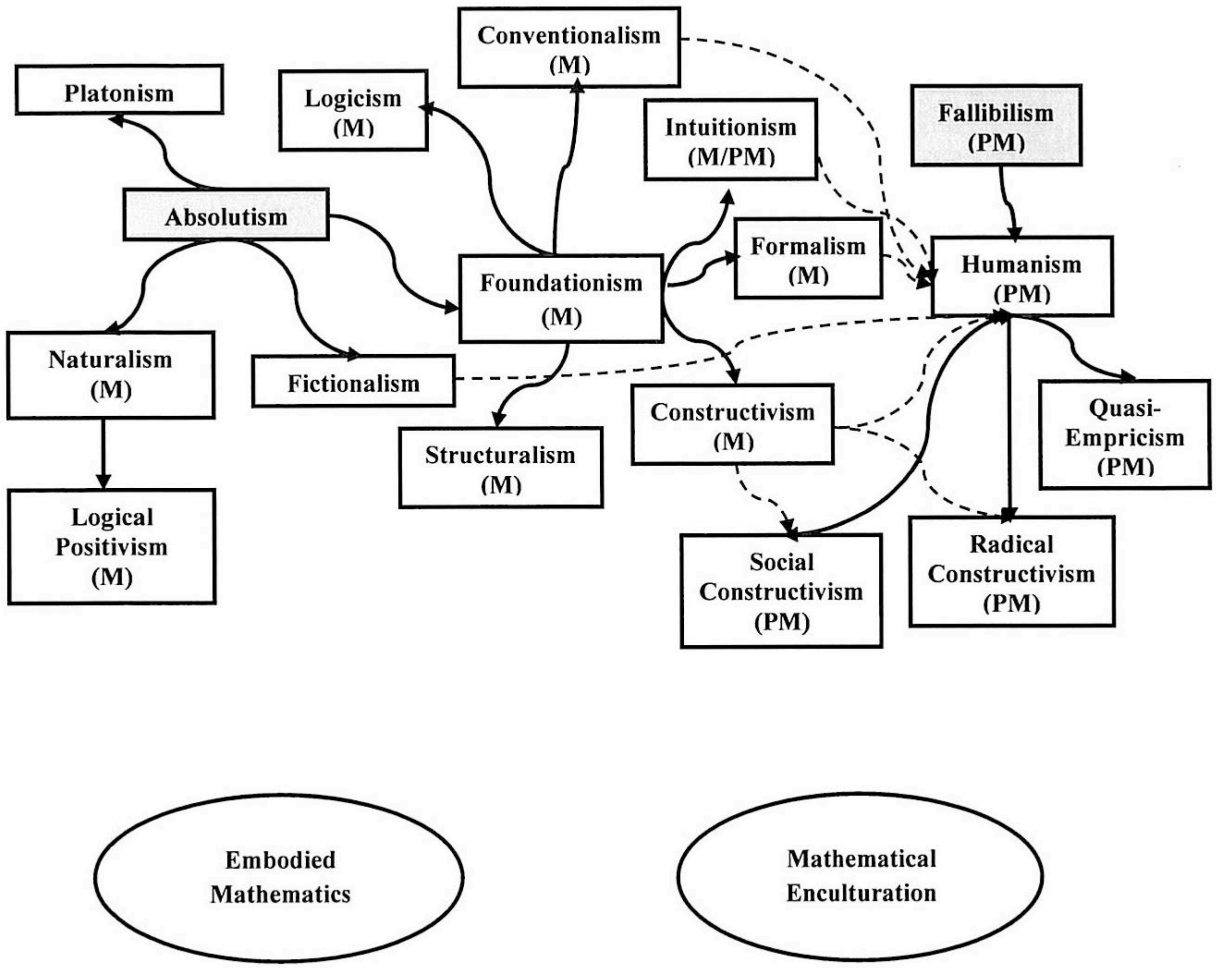


**Figure 1: Philosophies of mathematics, embodied mathematics, mathematical enculturation**

## Philosophies of Mathematics: Literature Review and Analysis

Many of the epiphanies in my story that were analyzed raise a different wording of my research question, namely, what is mathematics and consequently, how should it be taught and learned? Even the disparity between dialogues between my story and the two worldviews within that analysis present two very different conceptions of what mathematics is and how it should be taught and learned. It is this theme emerging from my story that I now explore in more depth, by turning to the literature that speaks to this very question: the philosophies of mathematics.

Philosophies of mathematics are concerned with the nature of mathematical knowledge, and thus also consider (albeit sometimes implicitly) the two questions (among others) of "what is mathematics" and "what is knowledge?" The varying answers to these two questions ultimately result in a variety of different philosophies of mathematics. It is also in the answers to these questions that proponents of alternate philosophies frequently find loopholes and paradoxes that, at least for them, render the philosophies under consideration ineffectual or obsolete. In the sections that follow, I will be explaining each of the many philosophies of mathematics that have been written about. During these explanations, I will endeavor to avoid the criticisms made by champions of one philosophy or another, but during the analysis of the philosophies I have no doubt that my comments will resonate with my own personal biases based upon my evolving stance with respect to a worldview. It is my belief that these biases will be evident to the reader and that my arguments will not be rendered irrelevant because of my openness in disposition. For interested readers, the specific arguments between and against the various philosophies of mathematics can be found, and quite elegantly in this regard, within the work of many other authors and researchers (e.g., Ernest, 1991; Hersch, 1997; Lakatos, 1978).

In his discussion of the philosophies of mathematics, Ernest (1991) categorizes the differing (past and present) philosophies of mathematics into two camps: the absolutists and the fallibilists. The difference between these two camps lies in their response to the notion of mathematical truth. For the absolutists, mathematics is "a body of infallible and objective truth, far removed from the values of humanity" (p. xi); moreover, "mathematics is the one and perhaps the only realm of certain, unquestionable and objective knowledge" (p. 3). For the fallibilists however, "mathematical truth is corrigible, and can never be regarded as being above revision and correction" (p. 3). Lakatos (1978) similarly categories the philosophies of mathematics, but uses the alternate names of Euclideans and quasi-empiricists for absolutists and fallibilistis, respectively.

Hersch (1997), alternatively, uses Kitcher and Aspray's (1988) classification system for categorizing different philosophies of mathematics: the mainstream, and the humanists and mavericks. In this schema, the categorizing criteria are based upon the relation of mathematics knowledge to humans. That is, the Mainstream philosophies are those that "see mathematics as superhuman or inhuman" while the humanist and maverick philosophies (and philosophers) "see mathematics as a human activity" (p. xiv). Although argued from a different perspective from Ernest and Lakatos, Hersch's categorization scheme results in the same split in philosophies.

Different nomenclature is frequently used in the naming of the various philosophies of mathematics, as is evidenced by Ernest's, Lakatos', and Kitcher and Aspray's alternate titles for the same categorizations. As well, even for one particular philosophy there are often many names used (for example foundationism is often equated with conventionalism, logicism, structuralism, constructivism and intuitionism). Moreover, certain philosophies of mathematics combine aspects (or could be seen as links) between two or more other philosophies that from a broader perspective might have seemed dichotomous (for example, intuitionism can be thought of as connecting absolutists and fallibilists through humanism). In essence, the philosophy of mathematics as a overarching picture is not precise or obvious, especially as one moves away (both historically and philosophically) from the original written philosophy of the Platonists and moves to the most recently conceived philosophies related to fallibilism and humanism (see Figure 2 below).

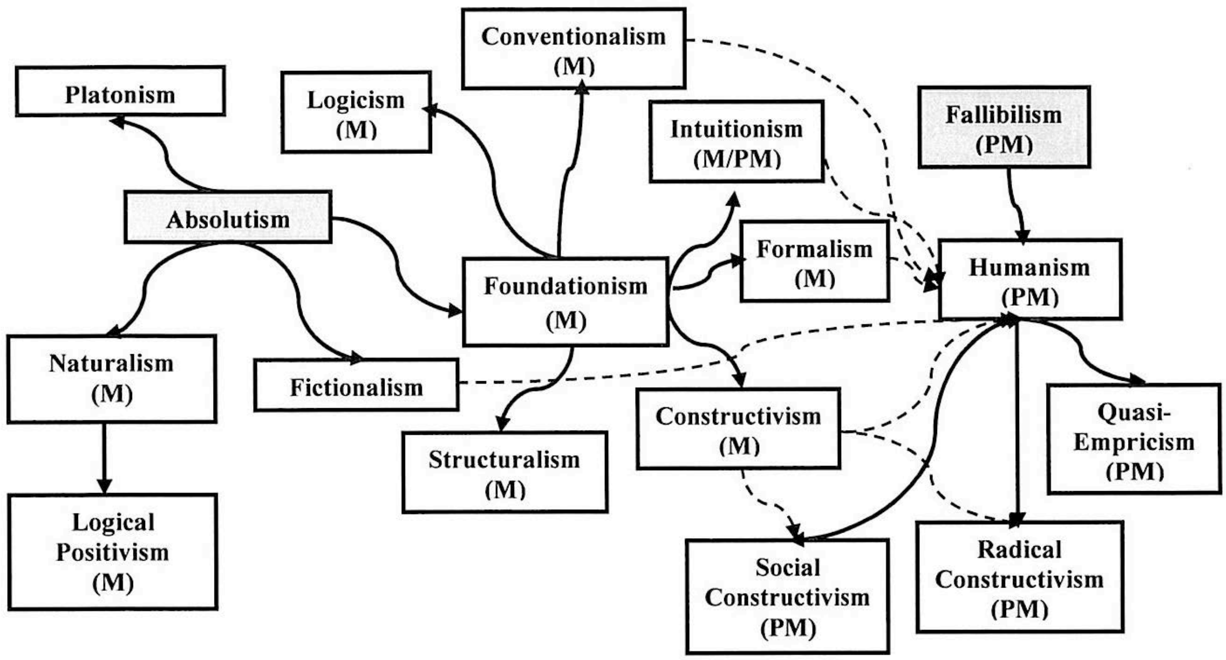


**Figure 2: Relationships between the philosophies of mathematics**

Generally speaking, the figure above represents a historical progression (moving forward in time from left to right) of the developments of the philosophies of mathematics, although many, if not all of the philosophies, do have ties back to the times of (for example) Pythagoras, Plato, and Descartes (making even the historical perspective of the philosophies of mathematics convoluted).

In my re-presenting of the characteristics of each of the philosophies of mathematics in Figure 1, I have chosen an new organizing classification of 'neither modern nor postmodern-like' (no label), 'modern-like' (M), and 'postmodern-like' (PM) to categorize the philosophies. In one case (intuitionism) both M and PM are given as labels because this philosophy of mathematics has features that are both modern and postmodern in nature. For the purposes of this paper, the classification of modern-like is given to those philosophies that are grounded in rationalism (the belief that knowledge is gained through reason), empiricism (the belief that knowledge is gained through the scientific method), and materialism (the belief in only a physical universe). Alternatively, philosophies in this paper that are classified as postmodern-like have characteristics that demonstrate an acceptance of ambiguity, paradox, disorder, differing approaches and methods, diversity, incommensurate interpretations, and skepticism (Molslehian, 2004). As is so often the case with classification systems, the one chosen for this paper leaves one philosophy of mathematics without a specific place to call home. This philosophy, embodied mathematics, is re-presented after the postmodern-like philosophies. In addition, although it is not a philosophy per se, Bishop's (1991) values of mathematics culture (mathematical enculturation), also warrant consideration, and as such are re-presented after embodied mathematics.

The numerous philosophies of mathematics are now presented in the specific categories described above (a very Traditional Western Worldview approach). Immediately following the descriptions of the philosophies of mathematics within a particular category, this data will be analyzed. As was the case in the analysis of my story, each category will undergo four different analyses: identification of prominent features or epiphanies in the data, discussion of what would be prominent within Gadamerian hermeneutic dialogues between the Traditional Western worldview and an Indigenous worldview and the data, and finally the coding and explaining of concepts emerging from the data using grounded theory. I begin by considering Platonism and fictionalism, the two philosophies of mathematics that are absolutist while also being neither modern-like nor post-modern-like in nature.

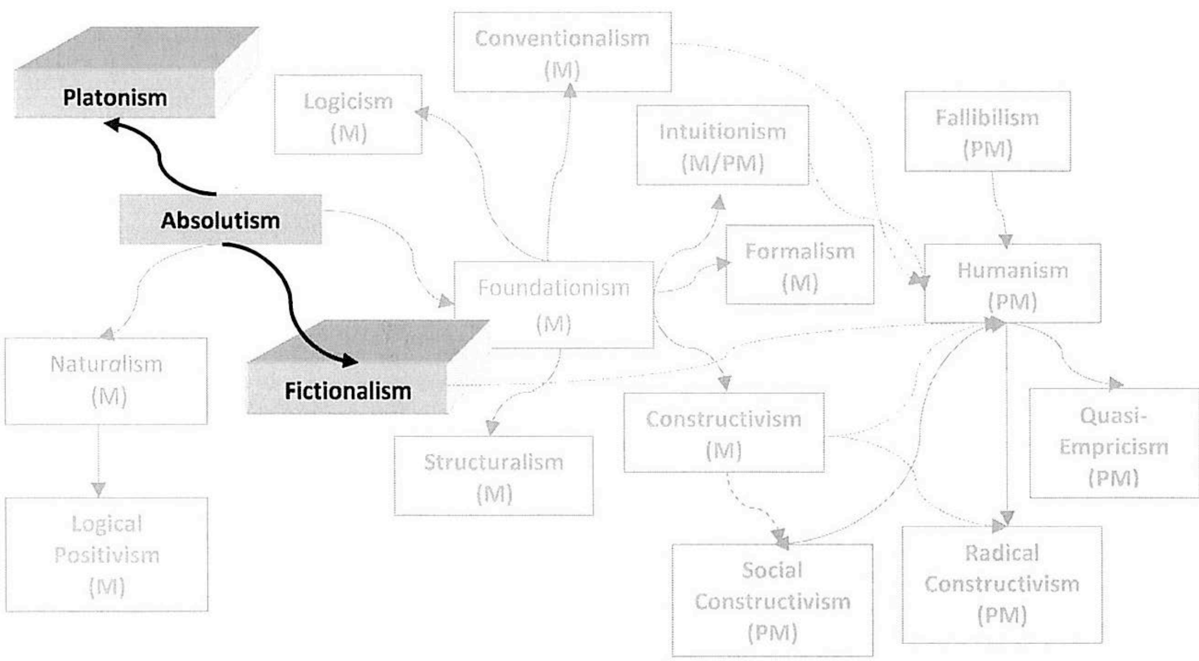

**Figure 3: Neither modern nor postmodern-like philosophies of mathematics.**

There are two philosophies of mathematics that are neither modern nor postmodern-like in nature: Platonism and fictionalism. Both of these philosophies are branches off of absolutism and therefore hold that mathematical knowledge is comprised of absolute and objective unquestionable truths. Each of these philosophies is now described in turn.

**Platonism**

The oldest (recorded in writing) philosophy of mathematics is Platonism, which is often referred to as Realism. It is also one of the most (if not most) common philosophies held by mathematicians and non-mathematicians alike today. Stemming from the work and beliefs of the Pythagorean's, and firmed up through Plato's writings (hence another common name for this philosophy being Pythago-Platonism), the main premise of this philosophy is that "mathematical entities exist outside space and time, outside thought and matter, in an abstract realm independent of any consciousness, individual or social" (Hersch, 1997, p. 9). Thus, "doing mathematics is the process of discovering [these entities] pre-existing relationships" (Ernest, 1991, p. 29). In its entirety, the Platonists view mathematical knowledge as being made up of descriptions of these outside entities, the relationships between them, and the structures that connect them to each other. As a consequence of this view of mathematics, complete objectivity is afforded to mathematics; truth exists in mathematics as it has its own inner laws and logic that it obeys to preserve the truth.

As well as the objectivity of mathematics in the Platonic philosophy, there is also the universality and absoluteness of the entities and their existence beyond the regular world of humans, like they are "members of the mathematical zoo" (Hersch, 1997, p. 11). In this philosophy of mathematics, "Every mathematical statement about number should be absolutely true or false. There should be no equally valid alternative forms of mathematics" (Lakoff, & Núñez, 2000, p. 80); "these objects exist outside physical space and time. They were never created. They never change" (Hersch, 1997, p. 11). Thus mathematics is construed as an infallible collection of knowledge that exists in "an independent, immaterial abstract world – a remnant of Plato's Heaven, attenuated, purified, bleached, with all entities but the mathematical expelled" (Hersch, 1997, p. 12), and having the distinct pleasure of being purely objective and always true.

Despite these commonalities in the view of where mathematics exists, and how it relates to truth and objectivity, there are different 'sects' of Platonism derived from alternate foundations for this superhuman knowledge. Two such examples are Thom's geometric grounding of mathematical ideas (i.e., the structure and relationships of the mathematical entities are grounded within geometric logic and laws) and Godel's set-theoretic grounding of mathematical ideas (in which the structure and relationships of the mathematical entities are grounded within the logic and laws of set-theory). Regardless of the foundation used to understand the mathematical entities, Graham's (a combinatorialist) sentiment: "I personally feel that mathematics is the essence of what's driving the universe" (as sited in Hersch, 1997, p. 11) is a seemingly natural conclusion based upon the infallibility and otherworldly nature assigned to mathematics by the Platonists' philosophy.

**Fictionalism**

The second philosophy of mathematics that is neither modern-like nor postmodern-like is fictionalism. For fictionalists, there does not need to be actual mathematical entities; "there can be representation without a represented" (Hersch, 1997, p. 20). A common example used to describe fictionalism, is that there is no real Mickey Mouse, despite there being many representations of Mickey Mouse (on film, on t-shirts, in theme parks). This view of mathematics comes from the perspective of other natural sciences, which make use of mathematics: "[fictionalists] try to show that science doesn't require… actual existence of mathematical entities. You can do science, they say, while regarding mathematical entities as fictional – not actually existing" (Hersch, 1997, p. 197). In this way, fictionalists argue that "Nonfiction corresponds to empirical science; fiction corresponds to mathematics" (Hersch, 1997, p. 180).

Fictionalism is in some ways similar to constructivism (to be discussed under modern-like philosophies of mathematics) and ultimately to humanism (to be discussed under postmodern-like philosophies), in that

> it is sometimes possible to account for features of a mathematical discovery by the interests, tastes and attitudes of the discoverer and sometimes also by the needs or traditions of his country.… the way in which it's thought of by its creators, mathematics is like an art such as fiction or sculpture. (Hersch, 1997, p. 140)

However, fictionalists do not stress the formal aspects of the construction of mathematical objects, nor are they concerned with the humanness of the activity.

Fictionalists are often described as materialists: "They notice that mathematics is imponderable, without location or size. Since only material objects, ponderable and volume-occupying, are real, mathematics isn't real" (Hersch, 1997, p. 180). Ultimately, in the philosophy of fictionalism, mathematics is a fiction, and thus although re-presented in this paper along with Platonism, fictionalism contradicts the premises of Platonism while remaining absolutist in nature.

**Analysis of the Neither Modern-like nor Postmodern-like Philosophies of Mathematics**

With the above understandings of Platonism and fictionalism, the two absolutist philosophies of mathematics that are neither modern-like nor postmodern-like, I will next proceed to the analysis of them. This analysis will first consider the prominent features, or epiphanies, with the discussion of the two philosophies, followed by a discussion of the responses of the two worldviews (the Traditional Western worldview and an Indigenous worldview) to the two philosophies, and then will end with coding of concepts that emerge from the three previous forms of analysis.

**Prominent features of the philosophies.**

It is important to first note why these two philosophies have been placed within their own category, as neither modern-like nor post-modern-like. Although Platonism does support the rationalism of modern philosophies, rationality and a structured rational method of creating knowledge is not a part of this philosophy of mathematics because mathematics is assumed to exist a priori to human reasoning and thinking. Whether someone has discovered a mathematical idea or element, that element already exists within its own abstract reality, and it will continue to exist, even if it is never discovered. Likewise, Platonism is not a postmodern philosophy of mathematics because there is no ambiguity associated with or allowed within mathematical knowledge. Within Platonism, mathematical knowledge is absolutely true and has absolute authority, without question.

Fictionalism, on the other hand, allows for a certain level of mathematical diversity across varying contexts, but within a context, all ambiguity is gone, making this philosophy not postmodern-like. As well, fictionalism does not depend upon absolute rationalism and reasoned approaches free of personal preferences, in fact it encourages the creators and users of mathematics to think about and use mathematics in ways that make the most sense to them individually. Thus, fictionalism is also not a modern-like philosophy.

These two philosophies of mathematics are based firmly upon the assumption that mathematics, and mathematical entities are not part of the reality that we live our daily lives within. Platonism and fictionalism differ from each other instead upon the question of whether mathematics and mathematical entities even exist, or need to exist. Whereas Platonists hold that mathematics exists in an abstract realm, completely isolated and independent of our reality and ourselves, fictionalists view mathematics as creative constructs whose existence is inconsequential.

As a result of the difference of opinion regarding whether mathematics is fact or fiction, reality or make-believe, the two philosophies also vary in their perceptions of the relationships between mathematics and the mathematical knower and user. For Platonists, mathematics is made up of absolute facts, and there is only one mathematics. Fictionalists, on the other hand acknowledge that mathematics is formed by the interests and needs of the user of the knowledge, that is, mathematics is formed and represented by individuals rather than being pre-determined in a singular form. It should be noted however, that this does not imply that fictionalists deny the absolute truth of mathematical knowledge, only that they believe that it can be thought of in many different forms.

**Dialogue with the Traditional Western worldview.**

Considering Platonism from the perspective of a person grounded within the Traditional Western worldview, the singularity, compartmentalization, abstraction, and authority that is assigned to mathematics and its entities would be viewed as reasonable and logical. Since there exists only symbolic representation of mathematics, the notion that true mathematics sits outside of the human realm would also be acceptable within the Traditional Western worldview, particularly as it puts more distance between the knower and the known. Consequently, the knowledge is made more rationally and logically based, and less likely to be interfered with by relationship and context.

On the other hand, from the perspective of the Traditional Western worldview, the notion that knowledge could be something that does not actually exist, almost spiritual in nature, as the fictionalists maintain, would be seen as highly suspect. Knowledge of value must be of an object external from oneself, but it must be of an object, not of something within one's imagination. Further, the notion that mathematics could be different for different people, depending upon their context and needs would be viewed as foolhardy by a person grounded within the Traditional Western worldview. Knowledge must be singular and abstracted to be of value, and there is only one way to come to that knowledge – the "right way". The Traditional Western worldview would, however, approve of the fictionalists belief in the absolute truth of mathematics, despite their insistence on making it responsive to the individual or context.

**Dialogue with an Indigenous worldview.**

In contrast to the responses to these two philosophies from the perspective of the Traditional Western worldview, an Indigenous worldview aligns better with fictionalism than Platonism. A person grounded within an Indigenous worldview would find the Platonists insistence upon mathematical knowledge having to be that of absolute truths with singular representations applying to all contexts because of its abstract form far too limiting upon the value of the knowledge. Even the Platonists' maintaining that mathematics and its entities must exist in some form beyond our reality would not provide enough diversity in thinking from the perspective of an Indigenous worldview, as Platonists would not allow this "otherworldly" existence of mathematics to be part of a spiritual, physical, or emotional realm. Mathematics' abstract reality within Platonism is strictly intellectually bound.

Fictionalism, on the other hand, would be more appealing to a person grounded within an Indigenous worldview. Within fictionalism, such a person would find openness, even responsiveness, to relationships between knowledge and knower and the valuing of context, both of which are of value within an Indigenous worldview. As well, by not really worrying about whether mathematics exists or how it might exist, fictionalism allows emotional, physical, spiritual, experiential, and intuitional knowledge to be considered and potentially valued. This potential for the acceptance of diverse ways of knowing and kinds of knowledge would be greatly valued within an Indigenous worldview. In fact, the only aspect of fictionalism that would be seen as limiting the kinds of knowledge that are valued from the perspective of an Indigenous worldview, would be its belief in the absolute truth of mathematics and its necessity in creating and applying scientific knowledge.

With understandings of how both of the two worldviews (the Traditional Western and an Indigenous) would respond to the Platonism and fictionalism, I now move on to a discussion of the concepts that emerge from the data and these analyses. As has been seen before, there is much repetition between the concepts that are present and how they are being interpreted and understood.

**Coding and explanation.**

Within these two, neither modern-like nor postmodern-like philosophies of mathematics, a number of the same concepts that emerged from my story likewise can be found. In particular, singularity, compartmentalization and isolation, abstraction, relationship and context all are significant (either for their encouragement or their denial) within Platonism and fictionalism.

Singularity as a concept is found in Platonism in the emphasis on the absolute truth of the mathematics and mathematical entities that are discovered and assumed to exist within the abstract realm of mathematical knowledge. Singularity, in terms of the "right way" of doing mathematics is also deeply embedded in Platonism. Within fictionalism, the notion of the "right way", both in terms of representing and doing mathematics, is challenged, and thus singularity in this sense is denied. However, fictionalists do uphold the singularity of truth of the mathematics, regardless of how it is represented or used.

Compartmentalization and isolation are also present in the two worldviews perspectives of the two philosophies of mathematics. From the Platonic perspective, mathematical knowledge is isolated in a different reality from the one of day-to-day life, as well as in terms of mathematical elements and knowledge being categorized within that alternate reality. Within fictionalism, compartmentalization of mathematics is based upon the context and needs of the particular user of mathematical knowledge. Therefore, the same mathematical knowledge could be categorized in different ways, sometimes isolated and sometimes embedded within other mathematics and other contexts.

The concept of abstraction is directly connected to the singularity and compartmentalization that has just been described. For Platonists, abstraction of mathematics begins in their identification of an abstract reality within which mathematics exists, and it continues in their viewing of mathematics as absolute truths that can be used in the solving of any problem. These truths must therefore be housed as abstract knowledge, capable of being applied at any time without change being required. Fictionalism, on the other hand, can be seen as questioning the authority of abstract mathematical knowledge, as it holds that it is fictitious knowledge that is created to meet the needs of the individual in their particular context. Abstraction of this knowledge is not required by fictionalists; they simply create the knowledge and use it as they see need to while they work with other knowledge that actually exists.

Likewise, the concept of authority and power is also present in both philosophies of mathematics. In Platonism, the authority and power of mathematics is situated within the absolute truth of the mathematical knowledge and its entities as well as within the isolation of mathematics from the everyday real world. In this way, the authority of the mathematics is not challenged by its relationships to a changing world. Mathematics sits both removed from the objects to which it is applied and in authority over them.

This notion of the authority and power of mathematics is also present in the philosophy of fictionalism, although its location (or lack there of) and its representation and use are not subjected to the same sense of singularity as found within Platonism. The purpose of mathematics for fictionalists is to give them the ability to work with other kinds of knowledges, and in that sense it is imbued with unquestionable authority and power.

Finally, the concepts of relationship and context are also central to the thinking behind both the Platonic and the fictionalist philosophies of mathematics. Specifically, from the perspective of Platonism, relationship and context are irrelevant in consideration of mathematical knowledge; in fact, relationship and context would be seen as unnecessary for, and even interfering with, the singular, abstract, compartmentalized and isolated, and authoritative nature of mathematical knowledge.

Fictionlism, on the other hand, values context and relationship because they allow an individual to construct mathematics in the way that they need and want it. Like a novelist, fictionalists create mathematical knowledge to suit the problem (story) that they are exploring and in doing so, establish relationships between how they represent and frame the mathematics they need and the context that they are inevitably applying it to.

Reflection upon these concept codes and the explanations provided leads to two important findings. First, with the exception of hierarchy and specialization, the concepts of relevance within the analysis of the first two of the philosophies of mathematics are the same concepts that emerged from the analysis of my story. Further, the elaborations of these concepts concur with those previously seen for the same concepts. In addition, similar merging and interdependence of the concepts to that seen in the analysis of my story are also present within the analysis of Platonism and fictionalism, leading to not only continued saturation of the individual concepts themselves, but also of the conceptual category noted previously.

With the completion of the analysis of the two neither modern-like nor postmodern-like philosophies of mathematics completed, the next category of philosophies will be presented and analyzed. In particular, the philosophies of mathematics to be considered are those that are modern-like in their positioning, while still remaining absolutist in nature.

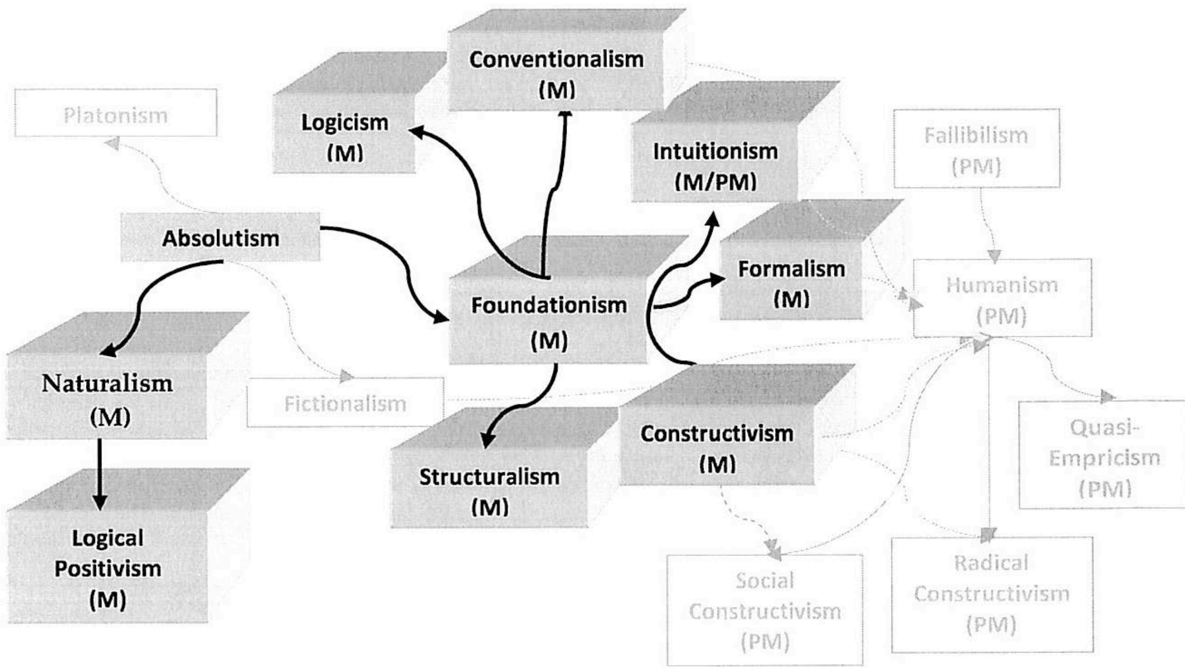


**Figure 4: Modern-like philosophies of mathematics**

The philosophies of mathematics in the modern-like category are naturalism, logical positivism (a subdivision of naturalism), and foundationism. Foundationism is subsequently broken into six subcategories: logicism, structuralism, conventionalism, formalism, constructivism, and intuitionism. With the exception of intuitionism, these philosophies of mathematics are grounded in beliefs that emphasize rationalism, empiricism, and/or the existence of only a physical universe, just as modernism does. Intuitionism does have this same grounding, but it also challenges the singular voice of modernism; therefore, I have chosen to label it as both modern-like and postmodern-like, with the emphasis being on the modern. Each of these modern-like philosophies is now discussed in turn.

**Naturalism**

A naturalist is "someone who rejects superstitious appeals to anything super- or non-natural, and rejects the conclusions of philosophical arguments when those conclusions conflict with what, on other grounds, it clearly appears rational to acknowledge" (Macbeth, 2001, p. 87). Moreover, naturalism is "the recognition that it is within science itself, and not in some prior philosophy, that reality is to be identified and described" (Quine, 1981, p. 21). As a consequence, in naturalism, mathematics' truths and knowledge are judged by scientific and rational methods.

Naturalism stems from absolutism, seeing mathematics as infallible objective truth, and it is so because it can be judged both scientifically and rationally. Like fictionalism, naturalism rejects Platonism, but for a different reason. Whereas fictionalism posited that mathematics was fictitious, and thus could not be otherworldly as Platonism suggests, naturalism outright denies the possibility of mathematics being beyond the natural world. With time, a branch of naturalism formed which narrowed the scope of how factual knowledge of mathematics could be determined and proved: logical positivism.

**Logical positivism**

Logical positivism (also known as logical empiricism) emerged as a philosophy of mathematics out of the perceived need to "find a natural and important role for logic and mathematics and to find an understanding of philosophy according to which it was part of the scientific enterprise" (Creath, 2013, para. 1). Logical positivists view scientific knowledge as the only factual knowledge humans have and, as such, the development of scientific knowledge depends upon experimental verification (such as formal proofs in mathematics) rather than upon personal experience.

Like naturalists, logical positivists reject the consideration of mathematical knowledge as being superhuman or inhuman, but the logical positivists are even more stringent in that they only accept scientific (including logic-based) verifications of mathematical truths. Thus, although also a branch of absolutism, logical positivism opposes both Platonism and fictionalism on the grounds of their acceptance and promotion of an inhuman and unreal nature of mathematics.

**Foundationism**

Foundationism (sometimes referred to as foundationalism) is the last branch of absolutism to be re-presented in this paper. Hersch (1997) argues that foundationism's roots "are tangled with religion and theology" (p. 91), but its ultimate goal "is to provide a systematic and absolutely secure foundation for mathematical knowledge, that is for mathematical truth" (Ernest, 1991, p. 4). The result is that "Since Dedekind and Frege in the 1870s and 1880s, philosophy of mathematics has been stuck on a single problem – trying to find a foundation to which all mathematics can be reduced, a *foundation* to make mathematics indubitable, free of uncertainty, free of any possible contradiction" (Hersch, 1997, p. 22).

In its original form, as foundationism has many branches of its own (to be discussed in following sections), the foundation was seen to be the real number system and the justifying of all mathematics, including this foundation, was central. "Foundationism," as coined by Imre Lakatos, has had many famous mathematicians amongst its followers: Frege, Russell, Brouwer, and Hilbert, to name a few. Many of these same mathematicians became more specific regarding the foundation that they believed to be correct grounding for mathematics, resulting in a number of branches for foundationism, including structuralism, logicism, formalism, constructivism, and intuitionism.

**Structuralism**

Structuralism is a branch of foundationism that defines "mathematics as 'the science of patterns'" (Hersch, 1997, p. 177), making patterns the foundation that this philosophy seeks to secure. The structuralists qualify this notion by arguing "not everyone who studies patterns is a mathematician. What about a dress-maker's patterns? What about 'pattern makers' in machine factories?" (Hersch, 1997, p. 178). As part of their philosophy, structuralists restrict the meaning of pattern, within the context of mathematics, to be a nonmaterial pattern which defines a mathematical structure, as opposed to something that is created on a piece of paper or metal. As both an advantage and disadvantage of the philosophy of structuralism, Hersch (1997) notes: "The structuralist definition fits mathematical practice, because it's all-inclusive. All mathematics easily falls under its scope" (p.179), as unfortunately does much other knowledge, which structuralists do not accept as mathematics.

**Logicism**

As indicated by its name, logicism as a philosophy of mathematics finds its foundation in logic. In fact, logicists views "pure mathematics as a part of logic" (Ernest, 1991, p. 8), where "'logic' is the fundamental laws of reason, of contradiction and implication – the objective, indubitable bedrock of the universe" (Hersch, 197, p. 147). Mathematical knowledge is knowledge which is grounded within logic.

Bertrand Russell, one of many notable mathematicians who are associated with logicism (others include Leibniz, Frege, Whitehead, and Camp), made two claims regarding logicism as a philosophy of mathematics. First, “All the concepts of mathematics can ultimately be reduced to logical concepts, provided that these are taken to include the concepts of set theory or some system of similar power,” and second, “All mathematical truths can be proved from the axioms and rules of inference of logic alone” (Ernest, 1991, p. 9). Logicists argue that “showing mathematics is part of logic would show it’s objective and indubitable” and ultimately redeeming “all mathematics by injecting it with the soundness of logic” (Hersch, 1997, p. 147). Unfortunately, Russell’s paradox, known colloquially as the Barber’s Paradox, at the very least, added a fly to the ointment that logicism offered as a philosophy of mathematics, hence Russell’s addition of “some system of similar power” to his two claims.

**Conventionalism**

In conventionalism, the foundation of mathematical knowledge and truth is viewed to be linguistic conventions. As such, conventionalists argue that “linguistic conventions provide the basic, certain truths of mathematics. Logic and deductive logic (proofs) transmits this truth to the remainder of the body of mathematical knowledge, thus establishing its certainty” (Ernest, 1991, pp. 30-31). Because of the prevalence of its use in conventionalism arguments, this philosophy of mathematics is often referred to as “if-thenism”, where the ‘if’ portion of mathematical statements provides the linguistic conventions, including meanings, which through deduction, result in the ‘then’ portion of the statement and mathematical certainty. For some scientists (and mathematicians), conventionalism served a utilitarian purpose: “in physics Poincare was a conventionalist. He thought it a matter of convenience which mathematical model one uses to describe a physical situation” (Hersch, 1997, p. 200). Thus Poincare, like others, appreciated the freedom that conventionalism afforded them through its grounding in the selecting and use of different linguistic conventions.

**Formalism**

Another branch off of foundationism is formalism. Formalism “is often condensed to two short slogans: “Mathematics is a meaningless game [where] ‘Meaningless’ and ‘game’ remain undefined” (Hersch, 1997, p. 7); and mathematics is a “meaningless formal game played with marks on paper, following rules” (Ernest, 1991, p. 10). Ernest further provides two specific rules for the game of formalism (of mathematics):

> 1. Pure mathematics can be expressed as uninterpreted formal systems, in which the truths of mathematics are represented by formal theorems.
> 2. The safety of these formal systems can be demonstrated in terms of their freedom from inconsistency, by means of meta mathematics (p. 10).

An interesting part of formalism is that rules need to be made; however, the making of rules does not have any rules, yet the rules themselves are never arbitrary: "The rules of language and of mathematics are historically determined by the workings of society that evolve under pressure of the inner workings and interactions of social groups, and the physical and biological environment of earth" (Hersch, 1997, p. 8). It is in this recognition of the role of society that formalism has ties to humanism (to be discussed under the heading of Postmodern-like Philosophies of Mathematics).

Hersch continues by noting that in the philosophy of formalism, "what mathematicians publish, cite, and especially teach, will decide the rules…. [The] rules are set by our consensus, influenced and led by our most powerful or prestigious members (of course)" (p. 9). Thus, the foundation for formalism is that "Mathematics is *axioms, definitions, and theorems* – in brief, formulas. A strong version of formalism says that there are rules to derive one formula from another, but the formulas aren't *about* anything. They're strings of meaningless symbols" (Hersch, 1997, pp. 138-139). Harkening back to fictionalism, in the formalist philosophy of mathematics, it is only when a formula is used in a physical context, giving it a physical interpretation, that the formula has meaning. At that point, the formula can be true or false, but "the truth or falsity refers only to the physical interpretation. As a mathematical formula apart from any interpretation, it has no meaning and can be neither true nor false" (Hersch, 1997, p. 139).

**Constructivism**

All of the branches of foundationism can be related to the ideas of the constructivism philosophy of mathematics, but they are fundamentally different based upon what is to be constructed first and how new constructions are to occur. "For constructivists knowledge must be established through constructive proofs, based on restricted constructivist logic, and the meaning of mathematical terms/objects of the formal procedures by which they are constructed" (Ernest, 1997, p. 11). Hence, constructivism has ties to logicism and formalism; however, it differs greatly by limiting what logic and formulas can be used – in particular, indirect proofs (such as proof by contradiction) are not accepted.

Constructivists work within their restrictions on logic to “reconstruct mathematical knowledge (and reforming mathematical practice) in order to safeguard it from loss of meaning and contradiction” (Ernest, 1997, p. 11). Thus, they focus on recreating mathematical truths and mathematical objects through constructivist methods, which are grounded in deductive proofs. For constructivists, “mathematics is the study of constructive processes performed with pencil and paper” (Ernest, 1997, pp. 11-12) and these constructive processes are the foundation that grounds mathematics and through which all mathematical knowledge is derived.

**Intuitionism**

Intuitionism, the last of the modern-like philosophies of mathematics, can be seen to have ties to absolutism, foundationism and constructivism in that it seeks a foundation upon which all mathematics can be constructed and deemed to be unquestionably true. In a seemingly humanistic way (to be discussed in the next section on postmodern-like philosophies of mathematics), intuitionism “acknowledges human mathematical activity as fundamental in the construction of proofs or mathematical objects, the creation of new knowledge” (Ernest, 1991, p. 29). Like constructivism, intuitionism looks to human endeavours for the creation of new mathematical knowledge.

What distinguishes intuitionism most from constructivism is that it seeks “secure foundations for mathematical knowledge through intuitionistic proofs and ‘ur-intuition’” (Ernest, 1991, p. 29), implying that “mathematics takes place primarily in the mind, and that written mathematics is secondary” (p. 12). This stance is contrary to constructivism (and all other philosophies of mathematics), which seeks to construct formal proofs within restricted constraints on the logic applied. Intuitionism, on the other hand, proposes that one’s intuition, and other informal methods, can and should be used to reveal more mathematical truths to help fill the gaps in the axioms of mathematical theory that intuitionists view as being “fundamentally incomplete” (p. 29).

Brouwer, a leader in the development of intuitionism, stated: “mathematics is founded on intuitive truths” (Hersch, 1997, p. 153). In so saying, Brouwer proposed a First and Second Act of intuitionism to define the premises and functioning of intuitionism in relation to mathematics. In the First Act, Brouwer describes intuitionist mathematics as “an essentially languageless activity of the mind having its origin in the perception of a move of time” (Hersch, 1997, p. 154).

This perception of a move of time is described as a moment when two distinct things are separated, with one becoming more prominent than the other; it is this separation and change in prominence "which is the basic intuition of mathematics" (p. 154). Brouwer describes this movement in time as the intuitive beginning of natural numbers, and explains that all other natural numbers (as well as all mathematics) can be constructed from this first intuition of movement in time (the number 1).

In his Second Act of intuitionism, Brouwer explains that new mathematical entities can be created from the resulting infinite sequences of mathematical entities that emerge intuitively from the original movement in time, and secondly through the properties of those entities that have already been acquired. These properties must obey the intuitionism condition that they hold for all mathematical entities that are "defined to be 'equal' to" (Hersch, 1997, p. 154) the original entity.

Intuitionism also calls into question the notions of truth in mathematics, suggesting that instead of 'true' and 'false', mathematical entities and properties should be classified as 'constructively true,' 'constructively false,' and 'neither'" (Hersch, 1997, p. 154). The removal of the dichotomy of mathematical knowledge being either true or false, but true, false, or neither, based upon how that knowledge is constructed, is the main departing point for intuitionism from being exclusively a modern-like philosophy of mathematics and taking on at least a hint of postmodernism.

**Analysis of the Modern-Like Philosophies of Mathematics**

With the above understandings of the modern-like philosophies of mathematics (naturalism, logical positivism, foundationism, structuralism, logicism, conventionalism, formalism, constructivism, and intuitionism), I will next proceed to the Gadamerian hermeneutics and grounded theory analyses of them. This analysis will again first discuss the prominent features of these philosophies, followed by a discussion of Gadamerian dialogue responses of the two worldviews (the Traditional Western worldview and an Indigenous worldview) to the philosophies, and then will end with coding of concepts that emerge from the three previous analyses.

**Prominent features of the philosophies.**

To begin with, it should be explained why these nine philosophies of mathematics have all been categorized as modern-like. Without exception, they all perceive mathematics as being based upon rationalism, empiricism, and materialism; mathematics is part of reality and it is known through logic and rational thinking. Intuitionism does stand out from the other philosophies in this section in that along with this modern-like stance, it also does accept a certain level of diversity and ambiguity, letting intuition guide individuals to potentially different approaches to and methods for defining the same mathematical ideas and concepts that are never considered absolutely true or absolutely false. For this reason, intuitionism has also been classified as postmodern-like; however, the limitations upon its postmodernist characteristics by its modernist foundations have resulted in its overall positioning within the modern-like classification of the philosophies of mathematics within my research.

The philosophies of naturalism and logical positivism are directly related to one another (as illustrated in Figure 4) because they both strictly hold that mathematical knowledge can never be super- or non-natural in origin. Likewise, both of these philosophies see mathematics as being comprised of compartmentalized, abstract and singular truths. As an offshoot of naturalism, logical positivism is more specific about the type of rational thought through which mathematical knowledge can be derived, and that is through experimental verification (such as mathematical proofs) and not just personal experience.

Within foundationism, and the six related philosophies of mathematics (structuralism, logicism, conventionalism, formalism, constructivism, and intuitionism), the securing of a strong foundation upon which all mathematical knowledge and ways of knowing can be built and stored is the primary focus. These philosophies, in general, still seek the rational knowledge favoured by naturalism and logical positivism, but they also aim to find an overarching organizing scheme upon which all mathematics can be built, housed, and preserved.

The focus of structuralism is on building mathematical knowledge based upon patterns. These patterns must necessarily be mathematical patterns, and not patterns that can be associated with trades or other human-dependent activities.

Logicism, instead, looks to formal logic for the grounding and organization of all mathematics. Logicists seek to logically deduce all mathematical knowledge thereby securing the mathematics within the logic used.

Conventionalism, on the other hand, looks to the language of mathematics, and in particular that of linguistic conventions within mathematics for the underlying structure of mathematical knowledge. These linguistic conventions are the way through which the logic and deductive truths of mathematics are disseminated, and thus conventionalists argue that these conventions form the backbone of the development and preservation of mathematical knowledge.

The next of the foundationism philosophies, formalism, argues that mathematics has a foundation of "*axioms, definitions, and theorems*" (Hersch, 1997, p. 138), and that these foundations are to be used to develop mathematics. Formalism is perhaps more allusive in its securing of mathematics because it seems to permit some sense of ambiguity by not saying what the rules are for using the axioms, definitions, and theorems, as well as by saying that the mathematics that results is a meaningless combination of symbols until it is applied. In either case, without physical interpretation or application, formalism argues that all mathematics is meaningless.

Constructivism sees the foundation of mathematics to be formal proofs that are used to construct new mathematical knowledge. Only certain kinds of mathematical proofs are accepted within constructivism based upon a restricted view of logic. In particular, indirect and other inductive proofs, are not permitted within constructivism. The goal of the use of deductive proofs in constructing mathematical knowledge is to ensure that the resulting knowledge is unquestionable. Thinking about, hypothesizing about, and even applying mathematical knowledge are not part of what is held to be true mathematics by constructivists.

Finally, intuitionism also argues that mathematical knowledge is constructed knowledge; however, intuitionists, such as Brouwer, argue that the foundation of mathematics is not the deductive proofs of constructivism; rather, the foundation of mathematics is intuitive mathematical knowledge. Intuitionists argue that mathematics emerges through intuition and other informal methods, thereby filling the gaps in the knowledge that already exists. By arguing that mathematical knowledge is personally constructed through intuition, this philosophy also contends that there are no absolute mathematical truths or falsehoods, only mathematics that is true, false, or neither in relation to the intuition of the person considering the particular mathematical notion.

Thus, although each of the foundationism philosophies seeks to ground mathematics within a particular foundation defining the kinds of knowledge and ways of knowing that mathematics is based upon, they differ significantly in terms of the kind of foundation chosen and the resulting limitations from that foundation choice. With these understandings of the modern-like philosophies of mathematics, I now turn to the results of using Gadamer's hermeneutic dialogues by considering how the Traditional Western worldview would respond to each of them.

**Dialogue with the Traditional Western worldview.**

From the perspective of the Traditional Western worldview, both naturalism and logical positivism seem reasonable in their valuing of mathematical knowledge and ways of knowing. Grounded within rationalism, and dismissing the super- and non-natural would be seen as the best option. Of these two philosophies of mathematics, logical positivism would be seen as the strongest because of its move towards clarification of exactly what kind of rational and logical thinking is required.

In general, the Traditional Western worldview would be in favour of foundationism and its related philosophies of mathematics because of their pursuit to define an underlying structure, a kind of hierarchy emphasizing the singularity and compartmentalization of mathematical knowledge, the "right way" to do and know mathematics. The response by this worldview to the specific foundations, however, needs to be considered in greater detail.

By eliminating consideration of patterns of design and other human-dependent activities, structuralism abstracts mathematics knowledge from the knower, and also defines a hierarchy of patterns and their perceived values. This underlying characteristic of the philosophy of structuralism therefore has strong ties to the Traditional Western worldview, both in terms of the focus on rational and logical patterns, and in the creating of a hierarchy of value for patterns.

Similar to structuralism, the Traditional Western worldview would consider the premise behind logicism one of value. By turning to logic, a very specific form of rational thought, and insisting on mathematics being grounded within logic and logically determined, logicism is valuing precisely a kind of knowledge and way of knowing that is also valued within the Traditional Western worldview. It is possible that someone grounded within the Traditional Western worldview might question not considering other forms of rational thought and reasoning with respect to mathematical knowledge generation; however, as it is logic being presented the "right way" to do mathematics, it is unlikely that this concern would be too loudly expressed. Instead, in the spirit of compartmentalization, those grounded within the Traditional Western worldview would likely put knowledge gained through other forms of rational thinking into a category of its own (not mathematics).

How a person grounded within the Traditional Western worldview would respond to the mathematical philosophy of conventionalism is an interesting question. Although the rational nature associated with the mathematics knowledge would be appealing to a person grounded within the Traditional Western worldview, the association of mathematics with language would require deeper reflection. After all, from the perspective of the Traditional Western worldview, language and mathematics should stand apart from each other, two different disciplines, two different categories. Closer inspection of conventionalism, however, reveals that the kind of language being centered within this philosophy of mathematics is a very particular kind of language, one that communicates mathematical logic and deductive proofs through symbols and other linguistic conventions specific to mathematics. With this specialization of the language being considered within the philosophy of conventionalism, all concerns would likely be alleviated, and the alignment with the Traditional Western worldview would be very strong.

From the perspective of the Traditional Western worldview, formalism's reliance upon axioms, definitions, and theorems would be firmly accepted. Presented as truths, these elements of mathematics would be rationally used to develop more mathematical knowledge of value. The lack of definite rules for working with the axioms, elements, and theorems, however, would be seen as somewhat concerning as would the notion that the abstract symbols and other notation is meaningless without a context. The ultimate conclusion, that without physical interpretation all mathematics is meaningless, would actually be of less concern, because within the Traditional Western worldview, knowledge is sought for the sake of knowledge only, not necessarily for how it might be used or what other roles it might play in the future.

Constructivism, perhaps more than any of the other modern-like philosophies of mathematics, strongly aligns with the Traditional Western worldview. The focus on formal (rational) proofs as the source of new mathematical knowledge in constructivism fits perfectly with the Traditional Western worldview's pursuit of logic-based and rational knowledge. Further, the exclusion of indirect proofs, because it is argued that their results remain questionable based upon assumptions that must be made, also aligns with the Traditional Western worldviews pursuit of absolute truth. Finally, the separation of mathematical thinking, hypothesizing, and applying what is viewed as mathematical knowledge within constructivism, confirms the Traditional Western worldview's categorization and isolation of knowledge, as does the defining of a hierarchy of kinds of knowledge and ways of knowing.

The last of the foundationism philosophies, intuitionism, poses the greatest challenge to the Traditional Western worldview. The less rigorous, less overtly rational approach to the creation of mathematical knowledge (through intuition) would be seen as focusing on knowledge that has very little value. Further, the denial of absolute truth and falsehood in relation to mathematical knowledge challenges the Traditional Western worldview's seeking of singular, abstract, and authoritative knowledge. Only the desire to fill gaps in mathematical knowledge within intuitionism would appear rational to a person grounded within the Traditional Western worldview, but given how that knowledge is to be created, the same person would not feel that this goal could actually be achieved through this philosophy of mathematics.

Thus, although there is much alignment between the modern-like philosophies of mathematics and the Traditional Western worldview, there are also divergences between them. As will next be discussed, the relationships between an Indigenous worldview and the modern-like philosophies of mathematics are also not straightforward.

**Dialogue with an Indigenous worldview.**

From the perspective of an Indigenous worldview, both naturalism and logical positivism would seem very limited in the kinds of mathematical knowledge and ways of knowing mathematics that are valued. The outright rejection of the super- and non-natural as sources of mathematical knowing runs contrary to an Indigenous worldview's valuing of spiritual, emotional, physical, and intuitional ways of knowing. Of these two philosophies of mathematics, logical positivism would be seen as the furthest removed from the values of an Indigenous worldview because of its specificity in relation to the kinds of rational and logical thinking that are valued. However, because an Indigenous worldview values rational and logical reasoning as part of the diversity of ways of knowing, it would not entirely reject these philosophies; instead, they would be seen as lacking in depth and conceptual value.

In general, an Indigenous worldview would be cautious in embracing foundationism or any of its related philosophies of mathematics because of their seeking to find a single, abstracted foundation for mathematical knowledge to be grounded in and built upon. This worldview would question the reasonability of assuming a "one size fits all" approach to thinking and doing mathematics as it restricts what is possible within specific contexts and in relation to individual people. With this understanding, however, it is still important to consider how an Indigenous worldview would respond to the individual foundations for each of the six philosophies.

The focus in structuralism on patterns as the foundation for mathematical knowledge would at first seem enticing to a person grounded within an Indigenous worldview; however, the further requirement that the patterns considered be restricted to rational and logical patterns that are not human-dependent (that is, they reside outside of the knower) would call this philosophy of mathematics into question. Within an Indigenous worldview, structuralism would be seen as privileging some kinds of knowledge while denying any value to others, and thus it would be regarded as having very limited value with respect to knowledge construction and use.

An Indigenous worldview's response to logicism would be similar to that of structuralism. Although it would concede that logic may well be one way to obtain mathematical knowledge, the notion that it is the only way to do so would seem far too restrictive to a person grounded within an Indigenous worldview to be of much value. The mathematical knowledge accepted within logicism would thus be accepted within an Indigenous worldview; however, further kinds of mathematical knowledge and ways of knowing would also be sought.

Alternatively, initially, a person grounded within an Indigenous worldview might be pleased to see the fusing of two categories of knowledge, mathematics and language, within the philosophy of conventionalism. Upon digging deeper into what specifically about language this philosophy considers to be foundational to mathematics, however, one soon realizes that a person grounded within an Indigenous worldview would question the need for the highly abstracted symbols and other linguistic conventions that are encouraged within conventionalism. Once again, an Indigenous worldview would have space within it for the philosophy of conventionalism, but because of the restrictions placed upon the language associated with mathematics by this philosophy, the kinds of knowledge and ways of knowing emanating from conventionalism would be seen as severely restricted and of limited value.

From the perspective of an Indigenous worldview, formalism presents many concerning limitations upon mathematical knowledge. The absolute truth and abstractness associated with the axioms, definitions, and theorems that are assumed to be the foundation of all mathematical knowledge would be very restricting upon the kinds of knowledge and ways of knowing valued within formalism. Likewise, the development of mathematical content knowing that unless it is applied the knowledge is meaningless opposes an Indigenous worldview's pursuit of knowledge for a purpose, for a greater good.

Likewise, constructivism, when viewed through the lens of an Indigenous worldview, would appear overtly constricting. Although an Indigenous worldview accepts formal proofs as a way of knowing, to restrict knowledge construction to only this way of knowing would be seen as eliminating the possibility of pursuing and finding knowledges which may be more relevant and valuable in certain situations. The further elimination of indirect proofs only increases the degree of limitations that would be seen within this philosophy by a person grounded within an Indigenous worldview. Finally, the isolation of proof from other mathematical endeavours, such as hypothesizing and applying, would further restrict the value of constructivism as a philosophy of mathematics from the perspective of an Indigenous worldview.

The last of the foundationism philosophies, intuitionism, is perhaps the most closely aligned with an Indigenous worldview. Intuitionism's acceptance of intuition, which may be rationally based, but need not be, would be viewed by a person grounded within an Indigenous worldview as allowing a greater diversity of knowledge and ways of knowing to be considered of value. An person grounded within an Indigenous worldview would have a concern, however, with intuitionists' pursuit of knowledge gap filling if those gaps were not related to knowledge that is actually needed. Seeking knowledge merely for the sake of filling in a gap that one can notice is not a valuable reason for seeking knowledge within an Indigenous worldview.

Thus, from the perspective of an Indigenous worldview, the modern-like philosophies of mathematics, although contributing some knowledge of value, would be seen to be mostly restricting the creation and sharing of knowledge and ways of knowing of mathematics that are of equal value. With the obvious differences between the two worldview responses to the modern-like philosophies of mathematics described, I now move onto the conceptual coding and explanation of the codes as found through out the modern-like philosophies of mathematics data and the responses from the two worldviews.

**Coding and explanation.**

Within this data set and the results of the dialogues between the two worldviews and the modern-like philosophies of mathematics, the concepts seen previously are again present. Each of these concepts is now discussed in relation to the modern-like philosophies of mathematics and their analysis thus far.

Hierarchy as a concept is quite prominent in many of the modern-like philosophies of mathematics, with there often being only two levels within the hierarchy: valued and not valued.

Within naturalism and logical positivism, hierarchy is present in the valuing of some knowledge (rational and logical) and the dismissal of other knowledge (super- and non- natural). Logical positivism takes this hierarchy one step by placing further restrictions upon how the rational and logical knowledge must be attained.

Similarly, all of the foundationism philosophies of mathematics are based on hierarchies that assume that mathematical knowledge needs to have a particular foundation to be of value. Consequently, any mathematics-related knowledge that does not come from that foundation is devalued. For example, within structuralism, the hierarchy is based upon knowledge of value emerging from patterns; however, not all patterns are assigned the same worth. Rather, material patterns are not valued, while nonmaterial patterns that define a mathematical structure are.

Logicism focuses instead on a hierarchy of logically derived mathematical truths; while conventionalism considers the abstract symbolic linguistic conventions to determine the value of mathematical knowledge. For formalists, knowledge of value must come from axioms, proofs, and theorems; however, within their hierarchy of knowledge of value, they explicitly state that it does not matter how these mathematical concepts are to be used in the construction of the knowledge. Furthermore, formalists do not differentiate between mathematical knowledge that is meaningful on its own and mathematical knowledge that is detached from specific meaning. The hierarchy present in constructivism, on the other hand, closely relates to that of logical positivism, differing only in that it further restricts what ways of knowing are of value (and hence what knowledge is generated) to exclude indirect proofs.

Intuitionism is perhaps the only one of the modern-like philosophies which does not emphasize a hierarchy of knowledge because this philosophy views mathematics as being based upon one's personal intuition and does not attempt to determine overall absolute truths. Thus, the only hierarchies that might be present within intuitionism are those placed by individuals upon their own personal knowledge.

Specialization, in particular, specialization in relation to ways of knowing, is also commonly present within the modern-like philosophies of mathematics. Most of the philosophies emphasize specialized approaches to the creation of mathematical knowledge, such as rational and logical thought, rigor and proofs, linguistic conventions, pattern recognition, and knowledge of axioms, definitions, and theorems. Only intuitionism and formalism might be argued to not require specialism because they allow for mathematics to be developed by individuals for individual purposes through methods that they see fit in order to fill gaps within mathematical knowledge.

Abstraction is also very prominent within all of the modern-like philosophies of mathematics. The overt emphasis that is placed on logical and rational thought (naturalism, logical positivism, structuralism, logicism, conventionalism, and constructivism), nonmaterial patterns (constructionism), linguistic conventions (conventionalism), deductive proofs (logical positivism, and logicism), and symbolism (formalism) requires abstract knowledge and ways of knowing. Only within intuitionism does the possibility exist that some less abstract way of knowing may enter into the creation of mathematical knowledge; however, the intuition that is sought and valued, in and of itself, is very often abstract in nature as well.

Singularity is also present throughout the philosophies of mathematics, as most, excluding again (somewhat) intuitionism and formalism, seek a "right way" for creating and using mathematics through the specializations mentioned above. Singularity is also discernable through the emphasis of many of the philosophies on absolute truth (intuitionism and formalism again being the exception).

The concept of isolation = also emerges through the modern-like philosophies of mathematics descriptions, but this time most notably through intuitionism and formalism. In intuitionism, the isolation of knowledge occurs because the knowledge is isolated to the knower and their intuitions; whereas, in formalism, the isolation of the knowledge emerges from the viewing of the abstract symbolism of the mathematics as meaningless and without rules to be followed. By assuming the knowledge to be meaningless combined with no preset rules as to how to use the knowledge, all of the knowledge that is created is naturally isolated from the rest.

Unlike in previous analysis sections, categorization and isolation do not necessarily work together within the modern-like philosophies of mathematics. In fact, other than in conventionalism, where two categories of knowledge (mathematics and language) are merged, there is no indication of any of the modern-like philosophies of mathematics having a concern with categorization of mathematical knowledge beyond knowledge of value and knowledge of less or no value. Once the knowledge is deemed valuable, the philosophies do not appear to attempt to further categorize it.

Relationship and context (story) are most conspicuous in relation to their almost absence from the modern-like philosophies. However, formalism does bring relationship and context into the discussion by positioning mathematical knowledge as meaningless when not being practically applied. Thus, within formalism, mathematical knowledge changes when it is in relation to a context. Likewise, intuitionism's acknowledgement of "constructively true," "constructively false," and "neither" also recognizes the role of context in the construction of mathematical knowledge. All of the other modern-like philosophies of mathematics seek knowledge in ways that would separate the knowledge from the knower, and thus eliminate the need, and even the desire, for mathematical knowledge that is related to context.

Finally, the concept of power and authority is also embedded within each of the modern-like philosophies of mathematics. Within most of these philosophies, the power and authority assigned to mathematical knowledge is directly connected to the hierarchies of knowledge of value that the philosophies embrace. Alternatively, for formalism and intuitionism, the power and authority of the mathematical knowledge is determined by the individual knower within their particular contexts.

Although the contexts through which these concepts have emerged are different from those that have been seen in previous analysis sections, the kinds of explanations remain consistent, contributing to the saturation of the concepts. Further, the undeniable links between different groupings of these concepts, such as between hierarchy, specialization, singularity, (categorization and) isolation, abstraction, and power authority, as well as the conflict that other concepts present to these mergers, such as relationship and contexts challenges to those same concepts, is indicative of the emergence of a conceptual category.

Moving on from the modern-like philosophies of mathematics, the next category of philosophies will be discussed – the postmodern-like philosophies of mathematics. All of the philosophies of mathematics within this category are fallibilist in nature, that is, they acknowledge what they perceive to be the imperfection of mathematical knowledge because of its dependence upon human intellect. The reader will note that these philosophies are provided in greater detail then those that came before. This difference is due to the availability of literature on them, and not due to a personal preference. Whether greater detail is provided in the literature because the people following the fallibilist philosophies of mathematics feel a need to justify their existence to the absolutists, because it is just the nature of taking a fallibilist stance that one feels the need to provided detailed explanations, or for some other reason I have not considered, I regardless chose to provide as much detail for all of the philosophies of mathematics that I could within in the literature review.

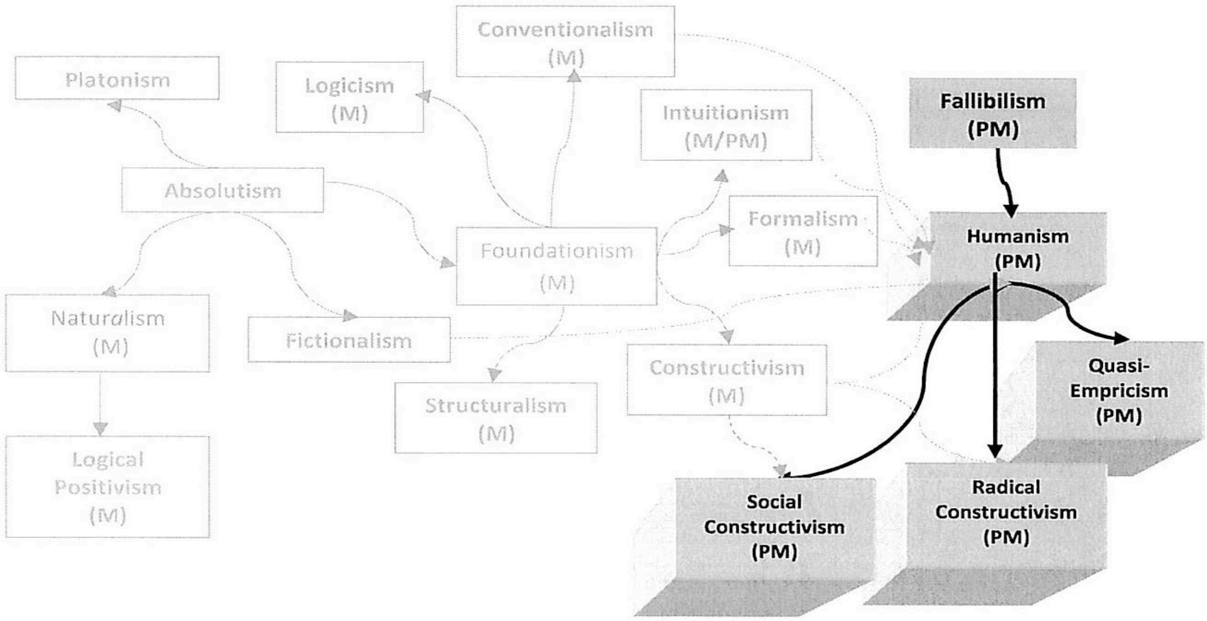


**Figure 5: Postmodern-like philosophies of mathematics**

Although not always made explicit in the philosophical theories, or not emphasized to the same extent as in the modern-like philosophies, the postmodern-like philosophies of mathematics, excluding intuitionism, which has both modern and post-modern characteristics, are fallibilist in nature. As noted earlier, the fallibilists are those who view mathematical knowledge as not being indubitable, rather subject to regular revision and correction. As a result of this fallibilist nature, mathematics cannot be categorically divorced from the empirical (and hence fallible) knowledge of the physical and other sciences. Since fallibilism attends to the genesis of mathematical knowledge as well as its product, mathematics is seen as embedded in history and in human practice. Therefore mathematics also cannot be divorced from the humanities and the social sciences, or from a consideration of human culture in general. Thus from a fallibilist perspective, mathematics is seen as connected with, and indissolubly a part of the whole fabric of human knowledge. (Ernest, 1991, p. 26)

The fallibilist philosophies of mathematics are postmodern-like in that they, in their own ways and to their own extent, all accept or tolerate ambiguity, paradoxes, differing approaches and methods, and diversity.  Fallibilism is foundational to all of the postmodern-like philosophies of mathematics, including humanism, quasi-empiricism, social constructivism and radical constructivism.

**Humanism.**

In defining the humanistic philosophy of mathematics, Hersch (1997) explains: “I use ‘humanism’ to include all philosophies that see mathematics as a human activity, a product, and a characteristic of human culture and society.  I use ‘social conceptualism’ or ‘social-cultural-historic’ or just ‘social-historic philosophy’ for my specific views” (p. xi).  In humanism, “mathematics must be understood as a human activity, a social phenomenon, part of human culture, historically evolved, and intelligible only in a social context” (p. xi).  As a result, Hersch argues that mathematics has no hidden meaning or existence beyond the use of it within the social-cultural-historic setting that the mathematics is situated within, explaining that “A socio-cultural-historical object exists only in some representation, whether physical (books, computer ‘memories’, musical scores, and recordings, photographs, drawings) or mental (knowledge or consciousness of people) or both” (p. 223).  Humanism views mathematics as a cultural, historical, and human activity – a much different stance from those of the absolutists.

Hersch (1997) further delineates four key features of the humanist philosophy of mathematics:

> 1. Mathematics is human.  It’s part of and fits into human culture.
> 2. Mathematical knowledge isn’t infallible.  Like science, mathematics can advance by making mistakes, correcting and recorrecting them.…
> 3. There are different versions of proof or rigor, depending on time, place and other things…
> 4.  Mathematical objects are a distinct variety of social-historic objects. They’re a special part of culture.  Literature, religion, and banking are also special parts of culture. Each is radically different from the others. (p. 22)

In so defining humanism, the characteristics that make it, at least partially, postmodern are readily visible – the accepting of different forms of validation, the fallibility of mathematics, and the recognition that mathematics and mathematical objects are socially constructed and reconstructed.

Arising from the basic premises of humanism are three other philosophies of mathematics: quasi-empiricism, social constructivism and radical constructivism.  Although the names “social constructivism” and “radical constructivism” would seem to relate these two philosophies more to the foundationism philosophy of constructivism, the similarity between these two humanistic philosophies and constructivism ends at the agreement upon the view that mathematical knowledge is constructed. Quasi-empiricism, social constructivism, and radical constructivism are now re-presented in turn.

**Quasi-empiricism.**

Unlike foundationalists, quasi-empiricists view the seeking of foundations for mathematics upon which all other mathematics can be developed as a pointless endeavor. Instead, the quasi-empiricists focus their philosophy on the methods of doing mathematics. Quasi-empiricism, as a philosophy of mathematics, emerged with the posthumous publishing of Irme Lakatos' final papers. Key to this philosophy are "three central themes… history, methodology and fallibilist epistemology" (Ernest, 1997, p. 116). Although none of the themes are original within the philosophies of mathematics, what is different is that all three are considered important within this one philosophy.

Ernest (1997) identifies Lakatos' major contribution to the history component of the quasi-empirical philosophy of mathematics as " his detailed case-study-based treatment of … the *logic of mathematical discovery*" (p. 117). The historical component of Lakatos' work explains the general cycle of methodology (the second theme) through which mathematics is developed, contested, and adapted:

> [it is] a cyclic process in which a conjecture and an informal proof are put forward (in the context of a problem and an assumed informal theory). In reply, an informal refutation of the conjecture and/or proof are given. Given some work on the topic, this leads to an improved conjecture and/or proof with a possible change of the assumed problem and informal theory. (p. 118)

Lakatos (1976) contended that "[foundationism] disconnects the history of mathematics from the philosophy of mathematics" in an attempt to ignore the uncertainty of mathematics, the existence of informal mathematics, and the reality of growth in mathematical knowledge. Thus, for quasi-empiricists, following Lakatos' lead, history plays a major role within their philosophy of mathematics as does defining the methodology of developing, accepting, rejecting, and adapting mathematical knowledge.

The third theme within quasi-empiricism, that of mathematics being fallibilist, or as Ernest (1997) writes "radical fallibilist" (p. 119), focuses on the necessity of informal mathematical theories when working with and on formal proofs. As fallibilists also argue, quasi-empiricists hold that no mathematical knowledge or object is definitively true; all mathematical knowledge is continuously subject to the possibility of revision or rejection.

Quasi-empiricists differ from other fallibilists in that within their mathematical methods, counter-examples as well as proofs are sought:

> Proof explains counter-examples, counter examples undermine proof … Each step of the proof is subject to criticism, which may be mere skepticism or may be a counter-example to a particular argument. Lakatos calls a counter example that challenges one step in the argument a 'local counter-example'; one that violates the conclusion itself, he calls a 'global counter-example.' (Hersch, 1997, pp. 211-212)

Quasi-empiricists actively seek both local and global counter-examples for new and old mathematical knowledge with the goal of expanding and continuously editing or rewriting that knowledge.

**Social constructivism.**

In social constructivism, mathematics is viewed as being constructed within social contexts and through social processes, and "is largely an elaboration and synthesis of pre-existing views of mathematics, notably those of conventionalism and quasi-empiricism" (Ernest, 1991, p. 42). In particular, social constructivists agree with conventionalists "that human language, rules and agreement play a key role in establishing and justifying the truths of mathematics" p. (42), with much emphasis being placed upon the role and use of language in the construction of mathematics. Social constructivists also support Lakatos' quasi-empirical approach to the methodology of mathematics in that "mathematical knowledge grows through conjectures and refutations, utilizing a logic of mathematical discovery" (p. 42).

Ernest (1991), a proponent of social constructivism, explains that it "is a *descriptive* as opposed to a *prescriptive* philosophy of mathematics, aiming to account for the nature of mathematics understood broadly as in the adequacy criteria" (p. 42). Ernest also defends the use of the word 'social' in the naming of this philosophy in three ways:

> (i) The basis of mathematical knowledge is linguistic knowledge, conventions and rules, and language is a social construction.
> (ii) Interpersonal social processes are required to turn an individual's subjective mathematical knowledge, after publication, into accepted objective mathematical knowledge
> (iii) Objectivity itself [can] be understood to be social. (p. 42)

It is to the third of the above defenses that social constructivists give the most attention. Social constructivists consider new mathematical knowledge, the creation of an individual, as subjective.  Once that new knowledge has been shared publicly (through publication or presentation to mathematician peers) and has withstood or been corrected by the social critiques involved in these processes, social constructivists say that the knowledge is then objective knowledge.  Both subjective and objective knowledge, as defined above, are accepted as mathematical knowledge by social constructivists and are the basis of a cycle of mathematical development and revision:

> new mathematical knowledge is from subjective knowledge (the personal creation of an individual) via publication to objective knowledge (by intersubjective scrutiny, reformulation and acceptance).  Using this knowledge, individuals create and publish new objective knowledge of mathematics, (Ernest, 1991, p. 43)

and thus the cycle continues with the creation and recreation of both objective and subjective knowledge.

In the critique process of the subjective knowledge, in its transition towards objectivity, social constructivists follow the cycle of mathematical verification as defined by Lakatos in quasi-empiricism.  Of course, social constructivism adds to this cycle that the criteria for refutation depends "to a large extent on shared mathematical knowledge, but ultimately they rest on common knowledge of language, that is, on linguistic conventions [the conventionalist view] of the basis of knowledge" (Ernest, 1991, p. 43).  Using the same definition and reasoning as above, social constructivists also hold that the linguistic conventions are objective because they are socially accepted.  The objectivity of the criteria for refutation strengthens the argument for the objectivity of published mathematical knowledge.

**Radical constructivism.**

In radical constructivism, as described by von Glasersfeld, knowledge is said to be constructed by individuals as a result of their repeated experiences and that it cannot be determined if that knowledge is representative of an outside "real" world.  In fact, "[von Glasersfeld] develops the provocative … proposition that all we can ever know about the real world is what the world is not" (Watzlawick, 1984, p. 14).  Thus, for radical constructivists, it is only when the result of applying one's knowledge leads to negative or contradictory results that you can be sure of whether your knowledge is a reflection of the real world, and at that point you know for sure is that the real world is not how you thought it was.  The radical constructivist believes the "real world" exists, but that is also unknowable.  As a result, for a radical constructivist, it makes no sense to talk about the existence of any particular part of the "real world."

In addition, radical constructivists hold that knowledge is not "discovered" but "constructed." "Radical constructivism, thus, is *radical* because it breaks with convention and develops a theory of knowledge in which knowledge does not reflect an 'objective' ontological reality, but exclusively an ordering and organization of a world constituted by our experience" (von Glasersfeld, 1984, p. 24). The result of this stance on knowledge is that coming to know mathematics does not result from passive receipt – rather it is created through active processes of the learning subject.

Radical constructivism also holds that the active construction of knowledge by a person not only takes place within one's experiences of the world, but that it is done so consciously to meet particular goals of that person: "The products of conscious cognitive activity, therefore, always have a purpose" (von Glasersfeld, 1984, p. 24). As to the question of whether the knowledge constructed is "good knowledge," radical constructivists do not attempt to develop a detailed process through which the "subjective" knowledge of the individual can become "objective" knowledge of the individual or a group. Instead, the test for what is good knowledge is how well that knowledge fits, or is viable for use, in a new experience. Radical constructivism is based upon the idea "that knowledge is good knowledge if and when it solves our problems" (von Glasersfeld, 1991b, p. 8).

Reflective of both Piaget's developmental theory, and Darwin's evolutionary theory, radical constructivism looks at the construction of knowledge by humans as the same in principle to that of all organisms: "Organisms live in a world of constraints. In order to survive, they must be 'adapted' or, as I prefer to say, 'viable.' This means that they must be able to manage their living within the constraints of the world in which they live" (von Glasersfeld, 1991b, p. 11). The knowledge of the subject must 'fit' with their new experiences.

> The substitution of the concept of fit (and its dynamic corollary, *viability*) for the traditional concept of truth as matching, isomorphic, or iconic representation of reality, is the central feature of the theory of knowledge I have called radical constructivism. (von Glasersfeld, 1991c, p. 64)

Thus, radical constructivism is focused on the applicability of mathematical knowledge rather than the absolute truth of knowledge.

Radical constructivists also hold that the processes of adaptation and assimilation are central to the construction of knowledge. Von Glasersfeld, via Piaget's developmental theory, posits that in the construction of knowledge, the cognizing subject seeks cognitive equilibrium (which is never actually reached):

> by assimilating the signals he is actually coordinating at a given moment (or stage) to the structures he has formed in the past; and he also works towards it by accommodating the already formed structures, whenever the signals with which he is operating cannot be fitted into one of the available structures as they are. (von Glasersfeld, 1991d, p. 85)

Thus, radical constructivism also views the construction of new mathematics as dynamically involved with previous mathematical knowledge.

Further, von Glasersfeld stipulates that radical constructivists are interested only in rational knowledge, although he does not dismiss what he terms "mystic knowledge" in general. In placing this limitation on the knowledge of interest for radical contsructivists, von Glasersfeld also turns to Manturana's description of the scientific method for direction in how the creation of such rational knowledge occurs, and that as a result: "once [a] belief has been established, there is powerful resistance against any suggestion of change" and a kind of "scientific tunnel vision" (von Glasersfeld, 1991e, p. 127). Along this same line, von Glasersfeld stresses that

> having constructed a viable path of action, a viable solution to an experiential problem, or a viable interpretation of a piece of language, there is never any reason to believe that this construction is the only one possible, (von Glasersfeld, 1991b, pp.12-13)

thereby emphasizing the fallibility of knowledge accepted within the radical constructivist philosophy. In fact, radical constructivism, itself,

> does not claim to be anything but a model, that is, a construct whose value depends exclusively on its viability. In other words, it will sink or swim according to whether it manages to establish and maintain equilibrium in the sphere of rational cognition. (von Glasersfeld, 1991a, p. 98)

This stance towards mathematical knowledge is significantly at odds with those of the previously discussed philosophies of mathematics, which emphasized the absolute truth of mathematical knowledge or the fortification of mathematical knowledge, or both. Radical constructivists do not seek to strengthen known mathematics; rather, they aim to evaluate it for its usefulness in an ongoing and case-by-case fashion.

**Analysis of the Postmodern-like Philosophies of Mathematics**

With the above understanding of the postmodern-like philosophies of mathematics (humanism, quasi-empiricism, social constructivism, and radical constructivism), I will next proceed to the analysis of them. This analysis will again first discuss the prominent features of these philosophies, followed by a discussion of the responses of the two worldviews (the Traditional Western worldview and an Indigenous worldview) to the philosophies, and then will end with the coding and discussion of concepts that emerge from the data and the three previous forms of analysis of it.

**Prominent features of the philosophies.**

Within the postmodern-like philosophies, the most prominent feature relates to how mathematical knowledge is seen as fallible, and therefore it is granted only limited power and authority. How the postmodern-like philosophies regard and respond to this fallibility (and thus what power and authority they attribute to the mathematical knowledge) is how the philosophies are most easily distinguished.

Humanism, and all of the postmodern-like philosophies begin from the stance that mathematical knowledge is the product of human activity. As such humanism views mathematics as a social and historically defined knowledge set that only makes sense within related contexts; beyond those historically defined social context, the mathematics does not exist or have any significant meaning. Humanism is postmodern-like because of its attributing of meaning and value of knowledge to time and place, as well as in its acceptance of different ways of knowing based upon those same conditions.

Quasi-empiricists argue that the humanists' mathematics, which is informal mathematical knowledge, can then be developed into formal mathematical knowledge. Specifically, quasi-empiricists actively engage in trying to provide examples of the fallibility of the informal mathematics by seeking counter-examples to it. Thus, within this philosophy, mathematical knowledge is put through a rigorous and cyclical process of conjecture and refutation. The conjectures are described as being proven through the use of informal mathematical knowledge, which itself is also caught in this cyclical process of scrutiny. Thus, quasi-empiricism acknowledges that mathematical knowledge can never be an absolute truth.

Within social constructivism, the social nature and components of the development of mathematical knowledge are the central focus within the cycle of conjecture and refutation of quasi-empiricism. Subjective and objective knowledge, which also play an important role within social constructivism, are defined quite differently from other uses of the words. Specifically, mathematical knowledge is deemed subjective knowledge when it is the knowledge of the person who created it. However, through the cycle of conjecture of refutation, and ultimately the social review, sharing, and publication, this subjective mathematical knowledge is said to become objective knowledge (accepted, until refuted, within the broader society). Through these processes, linguistic conventions are strictly adhered too, and those conventions are themselves considered objective knowledge, making the objective knowledge emerging from the conventions stronger.

Finally, radical constructivism emphasizes that mathematical knowledge, which is the product of an individuals repeated experiences, can not be claimed to be representative of the world of the knower. Instead, radical constructivism argues that the only thing that one can know about the real world is what it is not. Like quasi-empiricism and social constructivism, radical constructivism values a process of conjecture and refutation, but this time refutation comes through demonstrations of the knowledge not being viable within a particular context (any context). Thus, radical constructivism demonstrates a shift in philosophical thinking from the absolute truth of mathematical knowledge is to the viability of it. Radical constructivists do not argue that any particular mathematical knowledge is always true, rather that in particular cases it has proven to be viable. Thus, within this philosophy, mathematical knowledge is viewed as a model that can be tried within different situations, but it is also a model that is not guaranteed to work.

All of the fallibilist (postmodern-like) philosophies of mathematics focus on logical and rational knowledge and construction of knowledge. However, radical constructivism does acknowledge that "mystic knowledge" may exist, but such knowledge is not included within its consideration of mathematical knowledge and its viability.

Thus, although each of the postmodern-like philosophies of mathematics acknowledge the fallibility of mathematical knowledge, the reason why fallibility is attributed to the knowledge, and how that fallibility is responded to differs across the four philosophies (humanism, quasi-empiricism, social constructivism, and radical constructivism). With these understandings of the postmodern-like philosophies of mathematics, I now turn to the results of hermeneutically (grounded in Gadamer's theory) considering how the Traditional Western worldview would respond to each of them.

**Dialogue with the Traditional Western worldview.**

In general, the Traditional Western worldview would unequivocally reject all of the postmodern-like philosophies of mathematics on the basis that they are proposing the development and pursuit of knowledge that may (or even can) never be proven absolutely true. A person grounded within the Traditional Western worldview would appraise such knowledge as trivial and the pursuit of it pointless.

Specific to humanism, the Traditional Western worldview would also have difficulty accepting that mathematical knowledge is dependent upon social and historical conditions, as the social and the historical should sit outside of both each other and the mathematical. Furthermore, the restriction of such knowledge to contexts that are in relation to the social and historical conditions would be viewed as even more interfering in the determining of abstract knowledge of value.

At first glance, quasi-empiricism might be seen within the Traditional Western worldview as correcting the errors within humanism by proposing that the informal knowledge of humanism could be turned into formal mathematical knowledge. However, the dependence upon a cyclical process, for which there is no end, and thus no absolute conclusion once again throws this philosophy out of alignment with the Traditional Western worldview. This divergence between the worldview and the philosophy of mathematics then becomes a dichotomy, when the way that the informal knowledge moves to formal knowledge is described as being through contradiction rather than logical or rational proof and reasoning.

The issues that the Traditional Western worldview has with quasi-empiricism are somewhat compounded by the introduction of yet another layer of societal interference in the determination and validation of mathematical knowledge. However, a person grounded within the Traditional Western worldview would recognize some value in how social empiricism depends upon the specialization and knowledgeable authority of others. In order to move mathematical knowledge forward from the subjective (and thus inferior, from the perspective of the Traditional Western worldview) to objective, social constructivism requires the input of others who are recognized for their expertise in the particular mathematical area being explored, and thus it is acknowledging that some people have more authority in determining the "right" knowledge than others do.

Radical constructivism is possibly the furthest removed of the four postmodern-like philosophies of mathematics from the Traditional Western worldview in that it seeks viability and usability over abstraction and truth. The variability of mathematical truths across different contexts of use would seem absurd to a person grounded within the Traditional Western worldview because it does not provide a definitive answer, way of knowing, or even kind of knowledge that can be named as mathematics.

Thus there is far more divergence and disparity between the postmodern-like philosophies of mathematics and the Traditional Western worldview than has been noted with respect to any of the other philosophies that have been discussed. With this in mind, the relationships between an Indigenous worldview and the postmodern-like philosophies of mathematics will next be considered.

**Dialogue with an Indigenous worldview.**

Contrary to the Traditional Western worldview, an Indigenous worldview would be very accepting of the postmodern-like philosophies of mathematics stance that mathematical knowledge can never be proven or assumed to be absolutely true. A person grounded within an Indigenous worldview would agree with knowledge needing to be flexible and uncertain in order to be of value in all contexts.

The acknowledgement by humanism that mathematical knowledge is the product of human activity could both be accepted and questioned within an Indigenous worldview. Although a person grounded within an Indigenous worldview would be pleased to see that knowledge is being associated to human activity, the limited kinds of human activity, to that of rational thought would be seen as reducing the valuableness of the knowledge overall. Within humanism, there is no consideration given to emotional, physical, intellectual, experiential, or intuitional knowledge, nor to knowledge that is not for just the sake of *human* needs. An Indigenous worldview would, however, value how the philosophy of humanism attributes meaning and value to mathematical knowledge according to the context in which it is constructed and used.

As quasi-empiricism attempts to further formalize, even abstract, mathematical knowledge through the cyclical process of conjectures and refutations, an Indigenous worldview would also recognize limitations within this philosophy of mathematics knowledge pursuits. Instead of emphasizing the diversity of knowledge and ways of knowing that a person grounded within an Indigenous worldview would seek, that person would find that quasi-empiricism is actually attempting to narrow the diversity, to abstract its conclusions away from the specific context (by using contexts do dispute it). Since within an Indigenous worldview, knowledge is sought for the purposes of usability and giving back, spending time trying to refine and dispute knowledge when there is no obvious need other than the generation of more abstracted knowledge would likely be seen as time wasted. Thus, while the mathematical knowledge of quasi-empiricists would not be totally rejected within an Indigenous worldview, the overall value of this knowledge would be seen as narrow and lacking in overall significance to life.

The importance of social contributions to knowledge, and the social construction of knowledge within the philosophy of social constructivism, would alternatively be well received by an Indigenous worldview. Such recognition and seeking of social inputs to knowledge construction would be seen as an opening up to diverse ways of knowing and considerations. However, this same opening is narrowed substantially by the restriction placed upon the social membership, namely those perceived to have rational authority in relation to the area of mathematics being considered, thus again decreasing the value of this kind of knowledge (without eliminating it) within an Indigenous worldview.

The last of the postmodern-like philosophies of mathematics, radical constructivism, does make a strong tie to an Indigenous worldview in its argument that mathematical knowledge comes from an individual's experiences. Within an Indigenous worldview, the knowledge and experiences of an individual are greatly valued for the understandings that they contribute to the whole and to the particular situation. However, a person grounded within an Indigenous worldview would take exception to the notion that one can never really know the world, or something within the world. Instead, they would argue that an individual can have such knowledge, as it is the knowledge that they have. Moreover, that knowledge is not only based in logic and rationalism, but in other ways of knowing (such as emotional, physical, spiritual, cultural, and intuitional), which radical constructivism does not consider to be valid sources of mathematical knowledge. Alternatively, from the perspective of an Indigenous worldview, the valuing of knowledge for its viability within specific contexts within radical constructivism would be seen as appropriate.

Thus, from the perspective of an Indigenous worldview, the four postmodern-like philosophies of mathematics (humanism, quasi-empiricism, social constructivism, and radical constructivism) do hold some notions about mathematical knowledge and its construction that align well with the worldview. However, there are also many features to these philosophies which a person grounded within an Indigenous worldview would find very limiting, and thus would result in knowledge that is seen as not as valuable as it could have been.

Interestingly, both the Traditional Western worldview and an Indigenous worldview have more points of concern than alignment with any of the four postmodern-like philosophies of mathematics. With the understanding of these concerns and differences between the two worldviews and their responses to humanism, quasi-empiricism, social constructivism, and radical constructivism, I now move onto grounded theory's conceptual coding and explanation of those codes as they appear within this section of the data and the analyses of it so far.

**Coding and explanation.**

A very significant concept that emerges from the data and preliminary analysis of the postmodern-like philosophies of mathematics (humanism, quasi-empiricism, social constructivism, and radical constructivism) is undoubtedly the underlying notion of authority and power, and the tension that these philosophies have with the Traditional Western worldview around this concept. Since all four of these philosophies of mathematics are based upon an assumption of a fallibilistic nature of mathematical knowledge, they also all challenge any prior notions of mathematical knowledge carrying absolute authority and power. Although quasi-empiricism, social constructivism, and radical constructivism do propose a cyclical process through which the authority or power of mathematical knowledge might be strengthened, the three philosophies still maintain that the achievement of unconditional abstract truth can never be attained, and therefore, unquestionable authority and power can never be assigned to mathematical knowledge.

Despite the denial of absolute truth as well as authority and power, however, humanism, quasi-empiricism, social constructivism, and radical constructivism do still emphasize a hierarchy of knowledge. Humanism's presumed hierarchy is based upon its emphasis on rational and logic-based knowledge over other knowledges, and quasi-empiricism further delineates this hierarchy by putting more worth upon certain ways of knowing, and upon the knowledge progressing through the cyclical process of conjecture and refutation. The more cycles that a conjecture withstands through the process, the more valuable that knowledge is perceived to be, and the higher it's standing within the hierarchy of mathematical knowledge that it will be. Social constructivism, on the other hand, creates a hierarchy of what it defines as subjective (individual) and objective (societally approved) knowledge. Finally, radical constructivism places mathematical knowledge within a hierarchy of viability; the more useful a piece of mathematical knowledge shows itself to be, the more valuable it is perceived to be.

Specialization, on the other hand, only plays an explicit role within one of the four postmodern-like philosophies of mathematics, social constructivism. In the transitioning of the subjective knowledge of the individual to the societally approved objective knowledge, social constructivism turns to the part of the society that is comprised of individuals with specialized knowledge in relation to the mathematical knowledge proposed. Thus, the subjective knowledge is not assessed for it's potential or value by just anyone; instead, it is assessed by those who have been deemed (or possibly who have deemed themselves) as specialists capable of making these decisions.

Singularity, in fact the lack of singularity, within mathematical knowledge is also emphasized in all four of the postmodern-like philosophies of mathematics. This lack of a "right way" of knowing or doing mathematics is directly connected to the assumption of the fallibility of mathematical knowledge that is central to humanism, quasi-empiricism, social constructivism, and radical constructivism.

Categorization and isolation of knowledge also plays a less significant role within the four postmodern-like philosophies of mathematics, although it is still somewhat present. In quasi-empiricism mathematical knowledge is loosely categorized into "informal" and "formal" knowledge, dependent upon where in the cycle of conjecture and refutation the knowledge currently resides. Thus, the categorization of mathematical knowledge within quasi-empiricism can change depending upon the context and what has occurred previously. A similar form of categorization and isolation is also present within the philosophy of social constructivism. In this case, the categories are labeled "subjective" and "objective" knowledge; however, the premise is still the same – how a particular piece of knowledge is categorized depends where within the cycle of conjecture and refutation the knowledge is. Of course, how the change from one category to the other is different between the two philosophies; however, the kind of categorization and changes within categorizations are very similar. Also different from the categorization and isolation concept as it has appeared before is that, because of the assumption of the fallibility of mathematical knowledge, any piece of mathematical knowledge can be moved from one category to no category at all because it has been demonstrated to be incorrect.

Within radical constructivism, there are again two categories, viable or not viable. Being not viable ultimately equates with being shown incorrect within the philosophies of quasi-empiricism and social constructivism, so from that perspective, one might argue that radical constructivism does not categorize mathematical knowledge at all, just as humanism does not. Something is either mathematical knowledge or it's not considered within the philosophy.

Abstraction as a concept also appears differently within humanism, quasi-empiricism, social constructivism, and radical constructivism. Although the assumption of the role of logical and rational knowledge and ways of knowing within the construction of potential mathematical knowledge is present in all four philosophies, none of them assume the possibility of creating mathematical knowledge that is abstract from the contexts in which it is constructed and applied. This lack of abstraction is most notable within radical constructivism where the value of mathematical knowledge is directly associated with its viability in specific contexts and under specific conditions. There is no attempt within this philosophy of mathematics to seek or make claims to mathematical knowledge that can be applied to any situation, only to eliminate those that are not viable in particular situations.

Finally, the concept of relationships and context are also present throughout the four postmodern-like philosophies of mathematics. In quasi-empiricism and social constructivism, the relationship between conjectures and example, and the relationship between the individual and others in terms of knowledge production and valuing are central. Furthermore, radical constructivism places great importance upon the context in which particular mathematical knowledge is developed and in which it can be applied successfully.

Thus, the analysis of humanism, quasi-empiricism, social constructivism, and radical constructivism has again highlighted the same concepts that emerge from the analysis of my story as well as the analysis of the other philosophies of mathematics. During this particular part of the analysis so far, some new explanations and understandings of some of the concepts, such as for singularity and abstraction, have been provided. However, these new additions do not contradict previous understandings, rather they broaden, in the sense of the Gadamerian horizons of understandings, what is understood about those concepts.

Although with the presentation and analysis of the postmodern-like philosophies of mathematics, all of the (so far) identified philosophies of mathematics within the literature have been considered, there are two more views about mathematics and its origins that are worth considering: Lakoff and Núñez' (2000) embodied mathematics and Bishop's (1991) mathematical enculturation. These two notions explore what is going on beneath the surface of mathematical knowledge production – how the brain conceives of mathematics (Lakoff and

Núñez) and what the ideals and values of mathematics are.   Each of these ways of thinking about mathematics will next be presented, with each, in turn, undergoing the same analysis for prominent features, relationships to the two worldviews (the Traditional Western and an Indigenous) and the coding and explanation of concepts emerging from the literature presented and the other analyses of it.  I begin by considering embodied mathematics.

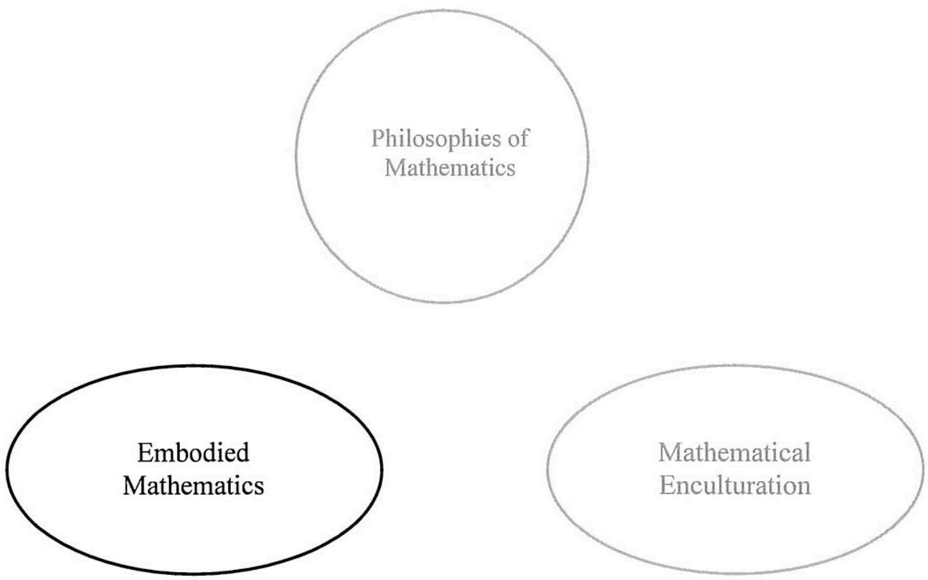


**Figure 6: Lakoff and Núñez's Embodied Mathematics**

Although conceived of as a cognitive learning theory, the work of Lakoff and Núñez (2000) and their study of the embodiment of mathematics through metaphors results in a completely different philosophy of mathematics that contradicts both the absolutist and fallibilist philosophies previously described. In *Where Mathematics Comes From*, the two researchers first report on a mythology, an almost romantic view of mathematics, that they encountered repeatedly in their research.  In particular, they found that it was commonly held that:

> Mathematics is abstract and disembodied – yet it is real.
> Mathematics has an objective existence, providing structure to this universe and any possible universe, independent of and transcending the existence of human beings or any beings at all.
> Human mathematics is just a part of abstract, transcendent mathematics. Hence, mathematical proof allows us to discover transcendent truths of the universe.
> Mathematics is part of the physical universe and provides rational structure to it.

> There are Fibonacci series in flowers, logarithmic spirals in snails, fractals in mountain ranges, parabolas in home runs and π in the spherical shape of stars and planets and bubbles.
> Mathematics even characterizes logic, and hence structures reason itself – any form of reason by any possible being.
> To learn mathematics is therefore to learn the language of nature, a mode of thought that would have to be shared by any highly intelligent beings anywhere in the universe.
> Because mathematics is disembodied and reason is a form of mathematical logic, reason itself is disembodied. Hence, machines can, in principle, think. (p. xv)

It should be noted that most of these characteristics, excluding possibly only that of disembodiment of reason, can be easily tied to one or more of the previously discussed philosophies of mathematics.

At the same time that they collected these myths, the two researchers "discovered that a great many of the most fundamental mathematical ideas are inherently metaphorical in nature", such as "The *number line* where numbers are conceptualized metaphorically as points on a line" (p. xvi). Along with these findings, Lakoff and Núñez (2000) considered recent research which had demonstrated that babies have inherent mathematical understandings, such as being able to distinguish between collections of two or three objects (including sounds) and knowing $1 + 1 = 2$ and $2 – 1 = 1$. Moreover, this knowledge was related to number (quantity) and not to any particular objects. Within just a few months, infants mathematical knowledge continues to expand to larger quantities (without intentional outside interference). By seven months, the infants are able to recognize if different collections of objects (or sounds) have the same quantity of elements.

As other mathematical abilities, such as subitizing (the recognition of "how much" at a glance), were being recognized within babies, infants, and young children, Lakoff and Núñez, like other researchers and authors (e.g., Butterworth), concluded that the human brain is wired for learning and doing mathematics. However, Lakoff and Núñez went further, tapping into cognitive science research and the use of metaphors, where metaphor is "not a matter of words, but of conceptual structure". As one of many examples they provide for a conceptual structure metaphor, 'affection' "is conceptualized in terms of warmth and disaffection in terms of cold" (2000, 41). This conceptualization creates a metaphor of the concept of affection, going well beyond a strict dictionary definition, and which is useful in interpreting and understanding how individuals understand the concept.

In the same way, Lakoff and Núñez (2000) have considered the question of what metaphors, if any, are foundational to the understanding of mathematics, and how those metaphors interact with each other as further mathematical knowledge is gained. Part of the metaphor building process is called coflation:

> the simultaneous activation of two distinct areas of our brains, each concerned with distinct aspects of our experience, like the physical experience of warmth and the emotional experience of affection. In a coflation, the two kinds of experience occur inseparable. The coactivation of two or more parts of the brain generates a single complex experience … It is via such coflations that neural links across domains are developed – links that often result in conceptual metaphor, in which one domain is conceptualized in terms of the other. (p. 42)

By linking physical and cognitive experiences, coflation results in embodied metaphors. Conceptual metaphors are made up of "a unidirectional mapping from entities in one conceptual domain to corresponding entities in another conceptual domain" (Lakoff and Núñez, 2000, p. 42). Thus, conceptual metaphors are seen to be part of the system of how we think, allowing us to "reason about relatively abstract domains using the inferential structure of relatively concrete" (p. 42) metaphors. Moreover, what is created are "metaphorical mappings [that] are systematic and not arbitrary" (p. 41). Lakoff and Núñez used these notions of metaphors, coflation, conceptual metaphors, and metaphorical mappings to try to understand mathematical knowledge and its creation.

An example of a common conceptual metaphor that Lakoff and Núñez describe as being part of mathematical knowledge is the container metaphor: "Categories are Containers, through which we understand a category as being a bounded region in space and members of the category as being objects inside that bounded region" (Lakoff, & Núñez, 2000, p. 43). The two researchers go on to say that the container metaphor explains why "the Venn diagrams of Boolean logic look so natural to us" (p. 45).

In addition, Lakoff and Núñez (2000) argue "that the 'abstract' of higher mathematics is a consequence of the systematic layering of metaphor upon metaphor, often over the course of centuries" (p. 47), resulting in a conceptual blend of metaphors for two distinct cognitive structures, such as when the properties of a circle are combined with the properties of the coordinate plane. If a metaphor is created to signify this conceptual blend, Lakoff and Núñez call the metaphor a metaphor blend.

In *Where Mathematics Comes From,* Lakoff and Núñez (2000) begin with metaphors for basic mathematical understandings and arithmetic, blending the resulting cognitive structures with further metaphors ,and ultimately working towards creating an understanding of mathematics as embodied knowledge. This view of mathematics is the foundation for what I term their philosophy of mathematics.

Contrary to the absolutist philosophies discussed previously, the philosophy of embodied mathematics holds that "mathematics, as we know it or can know it, exists by virtue of the embodied mind" (Lakoff and Núñez, 2000, p. 364). Moreover, the philosophy of embodied mathematics holds that all mathematics (known and not yet known) comes to being through "embodied mathematical ideas" (p. 364) of which many are "metaphorical in nature" (p. 346). In so positioning the origins of mathematical knowledge, Lakoff and Núñez step out of the philosophical arguments over whether mathematics is real-worldly or not and position mathematics as something internal to the human mind.

In response to the fallibilist philosophies, the philosophy of embodied mathematics argues that because mathematics is embodied. It "uses general mechanisms of embodied cognition and is grounded in experience in the world" (Lakoff and Núñez, 2000, p. 365), and as a consequence is not arbitrary. This conclusion results in the philosophy of embodied mathematics viewing mathematical knowledge as not purely subjective, not only the result of social agreement and not just dependent on history and culture.

The philosophy of embodied mathematics also addresses what mathematical objects are – embodied concepts – whether or not mathematical truths exist. In reference to truth, Lakoff and Núñez (2000) posit: "A mathematical statement can be true only if the way we understand that statement fits the way we understand the subject matter and what the statement is about. Conceptual metaphors often enter into those understandings" (p. 366). It is in this comment, that although they are arguing for different processes for the construction of knowledge than those proposed by radical constructivists, one can see that this philosophy of mathematics also looks for knowledge that fits, and one could argue is viable; in attempting to confirm the "good" of the knowledge.

Embodied mathematics, as a philosophy, is neither modern nor postmodern. Moreover, it has characteristics that are both absolutist and fallibilist, making it an outlier, yet a player amongst, all of the other philosophies of mathematics.

**Analysis of Lakoff and Núñez's Embodied Mathematics**

Based on the above understanding of Lakoff and Núñez' (2000) cognitive theory of embodied mathematics, I will next proceed to my analysis of it. This analysis will again first discuss the prominent features of embodied mathematics, followed by a discussion of the responses of the Traditional Western worldview and an Indigenous worldview to the theory, and finally will end with the coding and discussion of concepts that emerge from the above explanation of the theory and the other three analyses of it already done.

**Prominent features of embodied mathematics.**

Possibly most importantly, it should be noted how Lakoff and Núñez' (2000) theory of embodied mathematics, unlike the previously discussed philosophies of mathematics, is neither absolutist nor fallibilist in nature, that is it neither claims that mathematical knowledge is absolute, nor does it claim that it is fallible. Instead, this theory claims that mathematics knowledge just is, and that it is a metaphoric construction within the human mind. Without the human mind, mathematics does not exist, and because of the human mind it does.

The theory of embodied mathematics is also neither modern nor postmodern in nature. Although it does focus on rational reasoning and empiricism, the way in which mathematical knowledge is constructed according to Lakoff and Núñez (2000) is neither measurable nor really reproducible. Likewise, ambiguity, paradox, disorder, and diversity are not desirable in the metaphorical knowledge of embodied mathematics, rather the continual formation and reformation of metaphors and metaphorical networks aims to eliminate such issues as they appear.

Thus, Lakoff and Núñez (2000) address what mathematical objects are as embodied concepts without worrying about their truth, falsehood, or indeterminateness. Instead, what is of importance within their theory is that mathematical knowledge is based upon metaphors of understanding, and that these metaphors are themselves absolute and abstract from a particular context. However, each of these metaphors is in fact influenced by all contexts in which the mathematics is seen, thought of, or applied. The construction of mathematical metaphors within one's mind is thus a pursuit of best fit and ultimately viability. The metaphorical mathematical knowledge is thus not judged on its truth, but on how "good" it is within all contexts under consideration. As more and more contexts are considered, and modifications are made to the network of metaphors defining the mathematical knowledge, the "goodness" of the knowledge increases.

Overall, Lakoff and Núñez' (2000) theory of embodied mathematics not only does not align with absolutism, falibilism, modernism, or postmodernism – it considers mathematics from a very different perspective than that of the various philosophies of mathematics previously discussed and analyzed. Rather than trying to define what kinds of knowledge are valued in mathematics and what processes that a person needs to apply to obtain mathematical knowledge, embodied mathematics considers what mathematical knowledge looks like within the human mind and in particular how the mathematical knowledge is housed therein. With these understandings of the theory of embodied mathematics, I now turn to the results of hermeneutically (according to Gadamer's theory) considering how the Traditional Western worldview would respond to it.

**Dialogue with the Traditional Western worldview.**

When viewed through the lens of the Traditional Western worldview, the theory of embodied mathematics is, as has been seen many times before, somewhat murky. Although Lakoff and Núñez (2000) do restrict their focus to rational and abstract mathematical knowledge, which is strongly valued within the Traditional Western worldview, the possibility of emotional, spiritual, experiential, intuitional, or physical knowledge also contributing to mathematical knowledge cannot be discounted outright. For example, if one considers non-mathematical examples that Lakoff and Núñez provide (such as the metaphors of warmth and affection) such other knowledges are not completely disregarded in their theorizing about the embodiment of mathematics. Further, there is some valuing of experiences as informants to the construction of and relationships between mathematical metaphors, which would not be viewed as a reliable source within the Traditional Western worldview.

In addition, Lakoff and Núñez' (2000) embodied mathematics relies deeply upon the establishment of not only metaphorical relationships, but inter- and intra- metaphorical relationships as well. Therefore, the mathematical knowledge becomes so interconnected that complete isolation and categorization, or even the establishment of hierarchies of knowledge, would be next to impossible. This too would be challenged by a person grounded within the Traditional Western worldview.

Thus, a person grounded within the Traditional Western Worldview is very likely to view the pursuit of describing mathematical knowledge as metaphors unnecessary and whimsical dalliances, best saved for non-mathematicians to engage with if they so desire, but not to be held of importance in general terms. With these understandings of the relationships between embodied mathematics and the Traditional Western worldview, the relationships between an Indigenous worldview and theory of embodied mathematics will be considered next.

**Dialogue with an Indigenous worldview.**

From the perspective of an Indigenous Worldview, the exclusion of a discussion of alternate ways of knowing in favour of rational and logic-based approaches within the embodied mathematics philosophy would be seen as very limiting on the knowledge that one could attain mathematically. As well, the focus on creating the knowledge metaphors appears to be on the grounds that it can be done and not based on a need for it being done, which would also limit the value of it within an Indigenous Worldview. Further, although an Indigenous worldview would appreciate the emphasis on the relationships described and understood between different metaphors and metaphor networks, the use of abstract mathematical symbols to represent the knowledge contained within the metaphors would likely be viewed as making the knowledge restrictive. An Indigenous Worldview would, however, value the inter-relationships emphasized within Lakoff and Núñez's embodied mathematics and the recognition of the possibility of diversity in knowledge configurations between individuals' metaphor constructions.

In general, a person grounded within an Indigenous Worldview may view this philosophy of mathematics in much the same way as that of a person grounded within the Traditional Western Worldview – too focused on flights of fancy, but this time this view would be the result of the philosophy being too restrictive in terms of the kinds of knowledge and ways of knowing instead of too open, with too little concern for knowledge of value. Thus, embodied mathematics would be accepted as one possibility within an Indigenous worldview, but it would also be recognized as being knowledge that has limited value.

Thus, Lakoff and Núñez's (2000) theory of embodied mathematics does not have strong alignment with either the Traditional Western worldview or an Indigenous worldview. With this understanding, I now turn to the final analysis of this theory, that of the identification of significant concepts emerging from the theory and its analysis so far and an explanation of how those concepts apply.

**Coding and explanation.**

Within Lakoff and Núñez's (2000) embodied mathematics, there is no real emphasis placed upon a hierarchy of knowledge. It is true that within the metaphorical descriptions of different mathematics knowledges, metaphors may connect in a hierarchical fashion, requiring other metaphors in order to define a new one; however, there is no sense of one metaphor being perceived as having greater value than another as was prominent in both my story and many of the philosophies of mathematics.  Likewise, specialization is not even considered within this theory, other than to note that as people embody more mathematics, their conceptual metaphors become more specialized in their intent and purposes.

Embodied mathematics does bring some notion to the singularity of mathematical knowledge, as it is through specific relationships between specific metaphors that new mathematical ideas emerge.  Although Lakoff and Núñez (2000) never directly say that there is a "right way" in which these metaphors are defined and connected within our brain, their presentation of only one possibility in their discussion of the metaphor networks for different mathematical knowledges would seem to indicate that such an assumption would not be incorrect.

Categorization occurs within the theory of embodied mathematics through the grouping of metaphors into conceptual metaphors by Lakoff and Núñez (2000).  In their theory, the two researchers specifically target particular mathematical knowledge, isolated (at least at first) from other mathematical knowledge to be described through metaphors and the relationships between them.  As they progress with the construction of the conceptual metaphor, other metaphors (and hence mathematical knowledge) are brought in, but, only insofar as it is necessary for the development of the conceptual metaphor.  No attempt is made to further establish relationships between conceptual metaphors or to create broader categories of knowledge if not absolutely necessary.  In many ways, Lakoff and Núñez stick to the same categorization of mathematical ideas that is present within my story and through the various philosophies of mathematics.

Relationships, on the other hand, are a major concept within the theory of embodied mathematics; all knowledge is created by and related to metaphors.  Thus, within embodied mathematics, mathematical knowledge is the result of relationships between metaphors and how those metaphors relate to one's experiences.  Furthermore, the emphasis on personal experiences also emphasizes the importance of contexts within the creation of the mathematical metaphors. This connection, however, soon becomes irrelevant to the metaphor, which then stands apart from any specific examples.  At that point, the metaphor becomes an abstract representation of how the specific contexts impacted a person's mathematical knowledge, but the contexts are no longer retained.

The theme of authority and power is also present in Lakoff and Núñez's (2000) embodied mathematics, but in a different way from the philosophies of mathematics discussed so far. In embodied mathematics, authority and power is present within the individual, and not over other individuals or their knowledge, as each person constructs their metaphorical mathematical knowledge.

Overall then, the theory of embodied mathematics yet again brings forward the same concepts that have been seen throughout my research so far. Moreover, the interconnectedness of the various concepts, and how the presence or absence of one impacts others, is also evident, leading to further saturation of the individually coded concepts, while continuing to support an emerging axial coding of a conceptual category.

There now remains only one more piece of data to be introduced in relation to the philosophies of mathematics – Bishop's (1991) notion of mathematical enculturation. Bishop describes mathematical enculturation in terms of what he believes to be the values and ideals of Western mathematics. Inclusion of this data and its subsequent analysis should prove beneficial to broadening my (and the reader's) understanding of the kinds of knowledge and ways of knowing that are valued within mathematics and the teaching and learning of mathematics.

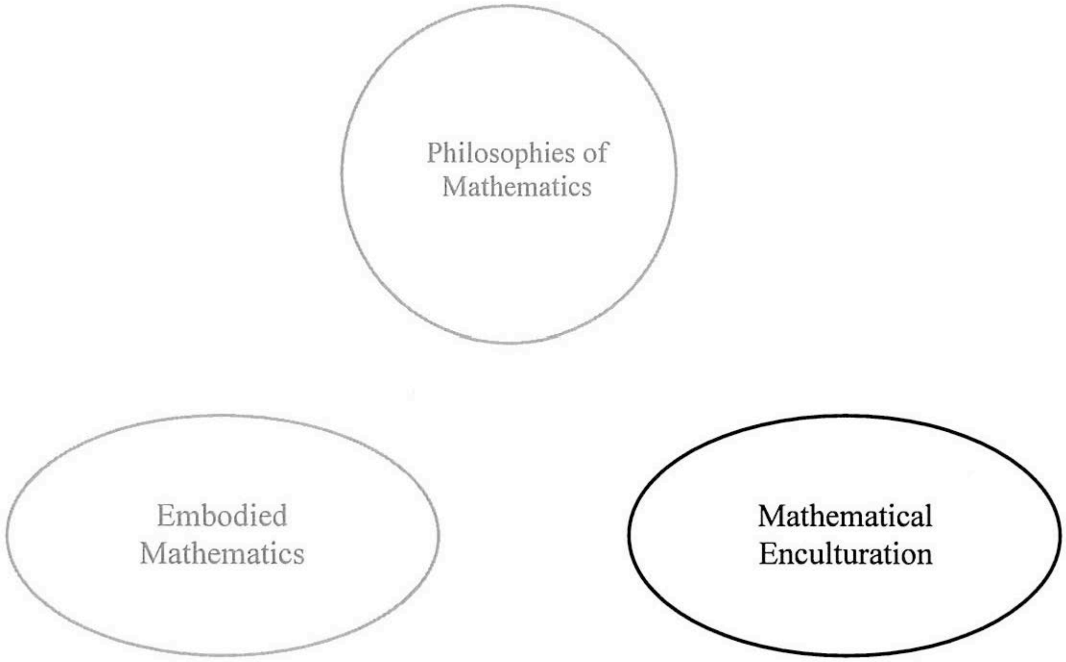


**Figure 7: Bishop's Mathematical Enculturation**

This final section in the re-presentation of the philosophies of mathematics, is, again, not on a philosophy of mathematics, but is a discussion of (Western) mathematics as a cultural product. This discussion comes from the work of Alan Bishop (1991) as provided in the book *Mathematical Enculturation,* in which Bishop recognizes that "mathematics is a pan-cultural phenomenon: i.e. it exists in all cultures" (p. 19), while also acknowledging that it recognizing that it could look differently amongst different cultures. Thus, in order to differentiate Western (abstract) mathematics (his primary focus) from other mathematics, Bishop refers to Western mathematics "as 'Mathematics' with a capital 'M'" (p. 19).

Of significance to this discussion is Bishop's (1991) presentation of the values of what he terms Mathematical culture. In particular, Bishop identifies "six different sets of ideals and values… in complementary pairs" (p. 61) that define Mathematical culture. These values are what Bishop holds to be "the principal values associated with Mathematics" (p. 62) in which students (and people in general) need to be enculturated. Each of these three pairings are now briefly discussed.

**Rationalism and Objectism**

The first of the pairings are the values of rationalism and objectism, both of which constitute the *ideology* of the Mathematics culture. Bishop (1991) defines rationalizing as "to seek to forge a logical connection between two ideas which may hitherto have been either unconnected or connected by incongruity" (pp. 63-64), and that it is only Mathematical explanations and arguments, not the world or people, which are rational. Within the rationalism of Mathematics, Bishop notes that although science and Mathematics were at one time concerned with explaining, Mathematics has moved away from empirical validation and is "concerned with 'internal' criteria of logic, completeness, and consistency" (p. 62). As a result, Mathematics is dependent upon deductive reasoning as the only means for "achieving explanations and conclusions" (p. 62).

When discussing rationalism, Bishop argues that it "has guaranteed the power and authority of Mathematics (and the ideal of Mathematicians)" (p. 62). Along with the ideological value of rationalism, Bishop also highlights its aesthetic nature "where the 'loose ends are tied up', where 'fuzziness' and imprecision are replaced by clarity and certainty, where greyness and shadowy half-truths are illuminated by the bright light of reason" (p. 64). For Bishop, "explanations are about abstractions, and these are the life-blood of Mathematics, as in proof, the pure form of Mathematical explanation" (64) making rationalism the heart of Mathematics. As a result, Bishop concludes: "Without understanding [rationalism], the language and symbols of Mathematics will be as meaningless to our children as are those of an alien culture" (p. 65).

The second value in this pairing Bishop (1991) calls objectism "in [an attempt] to characterize a world-view dominated by images of material objects" (p. 65). Like the value of rationalism, objectism is also bound to abstraction and removal of the personal being "based on inanimate objects and not on animate phenomena… Mathematics favours an objective, rather than a subjective, view of reality" (p. 66). Despite the efforts of rationalism to divorce Mathematics from the world and remain within the realm of ideas, those ideas themselves "originate in our interaction with the environment … it is material objects which provide the intuitive and imaginative bases for these ideas" (p. 66). Mathematics is, and relates to an, "objectivised reality" (p. 66). Thus, "the logical nature of Mathematics is complemented by its analogical side – its imagery – which is clearly rooted in society's world-view and in environmental interaction.  The imagery is object-oriented and materialistic, and can be a force for good or for evil in society, depending on one's view.  Given the significance of objectism in Mathematics, in relation to Mathematics education, Bishop argues that "as well as encouraging children to develop their ability to abstract [*rationalism*], we need also to encourage them in ways of concretising and objectivising abstract ideas" (p. 67).

**Control and Progress.**

Bishop's (1991) second pair of complementary values for Mathematics is control and progress, and are concerned with feelings and attitudes, or the *sentimental* component of Mathematical culture.  The value of control is seen in the "quest for knowledge, and explanations of natural phenomena,… a desire to predict" (p. 70), and the ability to predict is a powerful tool in maintaining control.  Such control gives humans "a sort of security within our ever-changing world" (p. 70).  As materialism became a driving force in the eighteenth century, there was development in "the understanding that Mathematics *can* explain any aspect of the natural or man-made environment, but there was also the growing *desire* to do this" (p. 70).  Bishop continues:

> it is interesting to see how we are not attempting to explain and control our (unknowable?) social environment through the development of social science.  The procedure is to try to understand human and social phenomena in Mathematical terms, in order to find rationally acceptable explanations of those phenomena and to help us end social 'problems'.  (p. 70)

Thus, through the sciences, Mathematics attempts to control the situations and objects in our lives.  Bishop also discusses how the "'facts' and algorithms of familiar Mathematics can offer feelings of security and control which are hard to resist" (p. 71), and how the solving of a complicated problem using those facts and algorithms can "kindle a glow of satisfaction and aesthetic pleasure" (p. 71).  The feeling of control through Mathematics soon spreads to the rest of society as it informs technological developments and other progress.  Bishop even argues: "As one progresses in Mathematics, the objects, the symbols, the rules become so familiar that they take on a certain kind of friendliness" (p. 71).

Complementary to the value of control within Mathematical culture, is the value of progress.  "Progress represents a more dynamic feeling than [control]" (Bishop, 1991, p. 72).  Bishop describes the value of progress as having "the feelings of growth, of development, of progress, and of change, and the first point of importance about this value is that the unknown can be known" (p. 72). Growth in knowledge is an aim of Mathematics, and thus it supports the value of progress Bishop proposes. Because such growth has been continuously achieved, it "is therefore felt to be continually achievable" (p. 72), which also results in a greater sense of control.

Bishop (1991) also explains how, when control is disrupted, such as when students encounter fractions or integers, and past knowledge (e.g., adding increases the quantity) is challenged, progress can then occur as the students learn that "because this is Mathematics, all this seeming chaos will be organized, structured, and thus explained, in such a way that the knowledge will once again offer security" (pp. 72-73). Bishop calls such an example one of personal progress.

Bishop (1991) then relates that progress can also be collective, such as when geometries other than Euclidean were proposed and validated. In this way, progress in Mathematical culture is about "alternativism – the recognition and valuing of alternatives" (p. 73). Bishop argues that today the embracing of alternativism is

> very strong – definitions, procedures, algorithms, axioms, proofs, are all capable of rich variation, and the exploration of alternatives is a powerful source of new research. In 'Western' society generally the spirit of alternativism seems to be alive and well, with alternate economies developing, alternative religions being studied and alternative lifestyles being pursued. (p. 73)

Desiring to emphasize that Mathematics is not value-free, Bishop also provides examples of how progress can in fact be detrimental if left unchecked: dissatisfaction with the amount of control one has over what is done in the environment, creation of unneeded technologies which results in the creation of an artificial need for them, and the creation of greater problems as a result of progress in the solution of a different problem. In this regard, Bishop ponders: "I wonder whether these values still have the emotional power to offer us an appropriate balance" (p. 75) – it is a problem that he leaves for education and educators to contend with.

**Openness and Mystery**

The final pair of values, openness and mystery, that Bishop (1991) discusses represent the *sociological* values of Mathematical culture. By openness, Bishop is referring to "the fact that Mathematical truths, propositions and ideas generally, are open to examination by all" (p.

75). In this way, “Mathematical principles, then are truths, as we like to think of them, namely open and secure knowledge. They don’t go out of date, they don’t depend on one’s political party, they don’t vary from country to country, they are universal and they are ‘pure; knowledge” (p. 75). As a warning however, Bishop comments that “it is important for this ‘purity’ that Mathematics is not about concrete, tangible objects… It is about abstractions which concern those tangible objects” (p. 75). Mathematics is thus depersonalized and in writing about their Mathematical findings, Mathematicians “conceal any sin that the author or the intended reader is a human being” (p. 75).

Mathematical openness also refers to Mathematical knowledge being available for “anybody to ‘own’” (Bishop, 1991, p. 75). Moreover, Bishop argues: “you can convince *yourself* that any Mathematical principle is true, nobody has to persuade you – ‘the facts speak for themselves’. Provided that you perform the correct procedures, and keep to the rules, logic will do the rest” (p. 76). This openness, Bishop contends, “reinforces and stimulates feelings of democracy and liberation within our societies and our social institutions” (p. 76) as within Mathematical knowledge:

> one is not a prisoner to tyrannical control, not forever at the mercy of gods who must be appeased, nor is one bound to certain people in authority. With rationalism as an ideology and progress as the goal, individuals are liberated to question, to create alternatives and to seek rational solutions to their life’s problems. (p. 76)

Thus, Mathematical knowledge is not only open within itself, but also can open the outside reality of individuals by providing them freedom and security.

Reflecting upon Mathematical culture, Bishop (1991) notes: “one of the paradoxes of Mathematics is that even though Mathematical culture brings with it the values of ‘openness’ and accessibility, people still feel very mystified about just what Mathematics is” (p. 78). For this reason, mystery is the last of the values of Mathematical culture that Bishop presents. This sense of mystery is not only felt by “the people-in-the-street” (p. 78), but by Mathematicians as well. In fact, Bishop claims that the mystery of Mathematics also includes a sense of mystery about Mathematicians, noting that often “we actually know some of their Mathematical products and ‘objects’ better” (p. 78) than we know the people who created them. This side of the mystery value within Mathematical culture is a natural consequence of the dehumanizing nature of both values of rationalism and openness.

Reflecting upon the history of Mathematics, Bishop (1991) notes that starting with the early Greeks, "'Abstraction' was necessary for the cultivation of Mathematics and … it also served to keep the Mathematicians abstract, remote, and exclusive", (p. 79). Further, the abstract objects of mathematics have no meaning for most people, although "professional Mathematicians who work with completely abstract phenomena as if they were objects will argue that these objects do have plenty of meaning for them" (p. 81). In relation to mystery as a value of mathematics then, Bishop contends: "'What is real?' … is destined to remain forever a mystery" (p. 81).

**Analysis of Bishop's Mathematical Enculturation**

Based on the above understandings of Bishop's (1991) value pairs related to Mathematical enculturation, I will next proceed to my analysis of it. This analysis will again first discuss the prominent features of the value pairs, followed by a discussion of the responses of the Traditional Western worldview and an Indigenous worldview to them, and then will end with the coding and discussion of concepts that emerge from the above explanation of Bishop's Mathematical enculturation and the other three analyses of it already done.

**Prominent features of mathematical enculturation.**

As in the postmodern-like philosophies of mathematics (humanism, quasi-empiricism, social constructivism, and radical constructivism), Bishop (1991) views mathematics as a human, even cultural, product and endeavor. Further, Bishop recognizes the potential for different mathematics and mathematical representations to exist within different cultural settings, and so to emphasize the mathematics he is focusing on in his discussions (and which he is most familiar with), Bishop chooses to refer to this mathematics (the mathematics of the Western society) with a capital M (Mathematics).

Bishop (1991) then delineates what he describes as the three pairs of values that determine what the nature of Mathematics is and how people engaging in Mathematics should understand it: rationalism and objectism (forming the ideology of Mathematics culture), control and progress (forming the sentimentality of Mathematics culture), and openness and mystery (forming the sociology of Mathematics culture). Each pairing of values represents a dichotomizing, yet intertwined perspective of aspects of Mathematics from Bishop's perspective.

When speaking of rationalism, Bishop (1991) is referring to the attempt within Mathematics to make logical connections between ideas, which focuses on the internal nature of Mathematical understanding. The objectisim of Mathematics is conversely the externalization of the knowledge into abstract constructs and representations. These two values, as is true within all the pairings, both need to exist in Mathematics, each supporting each other as they alternate in positioning themselves on centre stage.

Control and progress, take on a different perspective towards Mathematics, that being that they are concerned with feelings and attitudes in relation to Mathematics. Bishop (1991) uses the term “control” to explain how Mathematics is used to try to control the world around us, to explain and predict phenomena in order to be able to control them. Progress, on the other hand, is found in the dynamics of Mathematics, and Bishop uses the term to describe how one should seek to grow and change one’s Mathematical knowledge. By engaging in Mathematical progress, more Mathematical control can be sought and maintained, and progress emerges when control is interrupted. Bishop also argues that progress is grounded within the recognition and valuing of alternative strategies and knowledges.

The final of Bishop’s (1991) three value pairings foundational to Mathematics cultural is that of openness and mystery. Bishop uses openness to describe how Mathematical knowledge is open to anyone who wishes to pursue it. In addition, openness also refers to the security of Mathematical knowledge, that is, the consistency and dependability of the Mathematical knowledge. The mystery of Mathematical knowledge, on the other hand, is something which Bishop states is not sensed by everyone; however, non-Mathematicians do hold a sense of mystery in relation to Mathematicians, with Mathematicians rarely being associated in the general public with the Mathematics they have created and work with. Mathematicians, on the other hand, sense an additional kind of mystery in relation to Mathematics, that of what Mathematics might yet be created and how. Mathematical objects are often a source of mystery for both Mathematicians and non-Mathematicians alike.

Bishop (1991) introduced these valuing pairs in order to describe what people should know and understand about Mathematics as a discipline and subject area. In all instances, Bishop argues for balancing of emphasis within each of the pairings, rather than the disconnect and imbalance that he has often noticed during his time spent in engaging with others in Mathematics and discussions of Mathematics. With these understandings of Bishop’s three pairs of values of Mathematics, and the roles they play in the creation, use and, learning of Mathematics, I now turn to the results of hermeneutically considering how the Traditional Western worldview would respond to Bishop’s thoughts.

**Dialogue with the Traditional Western worldview.**

From the perspective of the Traditional Western worldview, although perhaps not Bishop's (1991) message, the identification and focus upon Western mathematics, and further the labeling of such mathematics as Mathematics, would be seen as valuable and desirable. Such a focus and granting of privilege to Western mathematics, which is already treated with respect and authority, aligns perfectly with the Traditional Western worldview's identification and valuing of the "right" knowledge.

In terms of Bishop's (1991) pairing of rationalism and objectism as the ideological values of mathematics, the Traditional Western worldview would question the need to build connections between Mathematical ideas in order to focus on the internal nature of Mathematics; since, the isolation of knowledge and not relationships between knowledges is an emphasis within this worldview. The Traditional Western worldview, however, would respond positively to Bishop's proposed objectism value because it focuses on the abstraction of Mathematical knowledge, a major goal within the worldview.

In the case of the pairing of control and progress, however, a person grounded within the Traditional Western worldview would commend both values. Control, as Bishop (1991) defines it, is about the authority and power of Mathematical knowledge, which aligns well with the Traditional Western worldview. Alternatively, Bishop's referral to progress aligns with the Traditional Western worldview's seeking of knowledge for the purpose of the knowledge itself. If that knowledge then leads to further control, a person grounded within the Traditional Western worldview would find even more reason to support these two values. Interestingly, however, the association of this pair of values with the sentimentality of Mathematics would not be well received from the perspective of the Traditional Western worldview, as it would imply that consideration should be given to an emotional, possibly even spiritual, nature of Mathematics. This is a nature that a person grounded within the Traditional Western worldview would, at best, choose to ignore because it would be seen as being of no value.

Finally, from the perspective of the Traditional Western worldview, Bishop's (1991) final values pair, that of openness and mystery, would be viewed as simultaneously appealing and objectionable. The security that Bishop's value of openness affords Mathematical knowledge would be seen to ensure the maintaining of the singularity and absolute truth that the Traditional Western worldview seeks. Moreover, the mystery that Bishop identifies as being afforded to

Mathematicians and their work would also appeal to a person grounded within the Traditional Western worldview since it emphasizes the notion of Mathematical knowledge being hierarchical in nature, with increasing levels of complexity, and thus resulting in a hierarchy of specialization and specialists as well. Some people can become those specialists, while others cannot.

Conversely, a person grounded within the Traditional Western worldview would reject the notion that because of value of openness anyone can do Mathematics, for if this was true, then there would be no need to value specialization and specialists. In addition, such a person would also not accept that parts of Mathematics are a mystery in that no one knows how to create them, or even recognizes that they are possible. While it is true that the Traditional Western worldview seeks knowledge for the sake of knowledge, it also assumes that it is possible to create or obtain all rational knowledge – none of it can, or would, remain hidden forever.

Thus, as seen in so many other analyses done thus far in my research from the perspective of the Traditional Western worldview, Bishop's (1991) values of Mathematical enculturation have both points of alignment and disagreement. With this in mind, I next discuss how Bishop's values of Mathematical enculturation would be (hermeneutically) supported or questioned within an Indigenous worldview.

**Dialogue with an Indigenous worldview.**

Probably the aspect of most importance with respect to how Bishop's (1991) ideas about Mathematical enculturation from the perspective of an Indigenous worldview, is how, by renaming Western mathematics, the mathematics that Bishop is focusing upon, as Mathematics, he is giving power and authority to that Mathematics over any other mathematics. This is done through the process of capitalization of the name, which changes it from noun to proper noun. From the perspective of an Indigenous worldview this is perhaps the first time that it could not accept, with or without reservations, any of the ideas about mathematics and the teaching and learning of mathematics that have been discussed this far. In so renaming mathematics, Bishop has in fact replaced everything that may have been considered mathematics (from any perspective) by a definitive subset of it, usurping all else in favour of itself. The only possible way for a person grounded within an Indigenous worldview to be able to accept this re-designation of Western mathematics to Mathematics would be to create a new category of knowledge that can accommodate all that Mathematics does not, but in creating a category of knowledge, this person would also be challenging their own worldview.

Moving beyond this label and onto the values of rationalism and objectism, a person grounded within an Indigenous worldview would find merit in attempting to find relationships between ideas, however, the insistence upon seeking logical connections only, would be viewed as very limiting of what is possible. Further seeking to externalize and abstract Mathematical knowledge would de-emphasize the importance of context and relationships, which are significant within an Indigenous worldview.

In terms of the values of control and progress, the divide between Bishop's (1991) values and an Indigenous worldview again widens, as an Indigenous worldview seeks to work in relationships with the world, not to exert power and authority over it. Moreover, an Indigenous worldview does not seek knowledge just to move knowledge forward, but because the knowledge being pursued is for the purpose of doing good for others as well as oneself, for contributing to community, family, and the cosmos. Thus, although it would not reject knowledge that is focused on control or progress only, an Indigenous worldview would find very little value in having only these goals for knowledge creation and sharing.

Finally, when Bishop's (1991) values of openness and mystery are considered through the lens of an Indigenous worldview, slightly more alignment appears. First, an Indigenous worldview values knowledge that is to be open to, and hopefully useful, for everyone regardless of who they are. As well, an Indigenous worldview would appreciate the acceptance of mystery, a kind of spirituality, within Mathematical knowledge. Further, this mystery might also relate to other ways of knowing, such as emotionally, physically, culturally, and intuitionally. In fact, the only real concern that a person grounded within an Indigenous worldview might have regarding these two values is that Bishop's notion of mystery could easily be tied to specialization and hierarchical viewing of Mathematical knowledge, both of which do not strongly align with an Indigenous worldview.

Thus, like so many times before with the philosophies of mathematics, Bishop's (1991) values of Mathematical enculturation do not overall strongly align with either the Traditional Western worldview or an Indigenous worldview. And so, I now turn to the final part of this analysis, that of the identification of significant concepts emerging from Bishop's values of Mathematical enculturation and the analysis of them so far, as well as providing an explanation of how those concepts apply to the ideas that Bishop has revealed.

**Coding and explanation.**

Yet again, the same concepts as previously identified and explained apply to Bishop's (1991) values of Mathematical enculturation. The only exception to this claim is that of context/story which could be assumed to play various roles within the value pairings, but which need not necessarily be present in order for Bishop's work to remain clearly defined.

The concept of hierarchy can be seen within Bishop's openness and mystery values. The first of these values, openness, challenges the notion of hierarchical knowledge by saying that Mathematical knowledge is available to everyone, while mystery acknowledges the hierarchical nature of Mathematical knowledge and the hierarchy of specialization between Mathematicians and non-Mathematicians, as well as between Mathematicians themselves.

The concept of the singularity of mathematical knowledge first emerges when Bishop claims the name Mathematics for Western mathematics alone. In so doing, he is, intentionally or not, moving Western mathematics to the top of a mathematical hierarchy, and is thus making Mathematical knowledge "the right", and therefore singular, knowledge. Singularity is also reinforced through the value pairs simply through the absence of the valuing of alternative approaches and the assigning of authority to those who have the specialized Mathematical knowledge.

Categorization and isolation are not as easily identified within Bishop's (1991) values of Mathematical enculturation, but are both present when Bishop shifts the view of his values from all of mathematics to specifically Mathematics. Each of the value pairs also define different categories of Mathematical knowledge: connected and abstracted (rationalism and objectism respectively), authoritative and dynamic (control and progress respectively), and accessible and restricted (openness and mystery respectively).

Relationship as a concept is actually a main foundation of Bishop's (1991) values of Mathematical enculturation, as he argues for a balanced relationship within each pairing in terms of how people view mathematics and how students learn about mathematics. Each of the pairs of values are also at once dichotomized and in relationship with each other, bringing a sense of complexity to the concept of relationship.

Abstraction is also present within Bishop's discussions of the values that he attributes to Mathematical enculturation. Most directly, abstraction is central to the value of objectism, but it is also through abstraction that both control and progress are obtained. Moreover, it is the abstraction of knowledge that Bishop argues makes Mathematical knowledge most accessible to everyone, and that often results in the mystery of mathematics as well. Like the concept of relationship then, abstraction as a concept is also being presented as a complex entity.

Finally, power and authority as a concept appears throughout Bishop's (1991) values of Mathematical enculturation. The capitalization of mathematics to represent Western mathematics clearly designates a power and authority to Western mathematics that is not attributed to other forms of mathematics. Further, the pairs of control and progress are inherently based upon the power and authority that Mathematical knowledge can exert on our lives. Even the values of openness and mystery link to the concept of power and authority, as openness is the value of Mathematical enculturation that gives everyone access to the power and authority of Mathematical knowledge; while mystery stands in the way of many individuals having access to the power and authority that Mathematical knowledge can afford to them.

Within Bishop's (1991) description of Mathematical enculturation, almost all of the previously identified concepts are again present, and even the absence of context (story) within this particular set of data does not eliminate it as a concept of importance within my research, rather it suggests that context may be an even more significant concept because of its overt inclusion and then exclusion from different data sets. Furthermore, there are again instances in which relationships between the various concepts suggest the formation of a broader conceptual category through which to reflect upon the data and to inform potential theory related to my research question.

**Final Reflections On the Philosophies of Mathematics, Embodied Mathematics, and Mathematical Enculturation**

Overall, my (Gadamerian) hermeneutic consideration and analysis of data related to the philosophies of mathematics and the related notions of embodied mathematics (Lakoff and Núñez, 2000) and Mathematical enculturation (Bishop, 1991) have produced similar results as did the analysis of my story. Throughout this section of my research, the philosophies of mathematics, as well as embodied mathematics and Mathematical enculturation, have produced various relationships with the Traditional Western worldview. At times, strong ties have been made with this worldview; however, more frequently the Traditional Western worldview has rejected foundational aspects of these ways of thinking about mathematics and mathematical knowledge. On the other hand, while not outright rejecting what is being proposed about mathematical knowledge and the teaching and learning of mathematics, an Indigenous worldview has been found to frequently devalue what is being proposed because it restricts the kinds of knowledge and ways of knowing to an extreme.

With respect to the grounded theory analysis of the philosophies of mathematics, embodied mathematics, and Mathematical enculturation, there has been a continuation and expansion of the understandings of the concepts that were identified in my initial analyses of my story. On a few occasions, particular concepts did not appear within a particular part of the analysis of this second data set; however, there was no consistency in terms of which concepts were not present. Moreover, when the concepts did not appear, it most often spoke to a broadening of the horizon of understanding of the concept that has been emerging throughout the course of all of the analyses. Overall, the continued repetition of both the concepts and continuing confirmation of their meaning is pointing towards their saturation.

In addition, as mentioned throughout most of the coding and explaining of the concepts, the individual concepts regularly inform the understanding and even existence of other concepts. Through these mergings and influences, the explanations of the concepts have been suggesting the emergence of a single conceptual category that speaks to and about what mathematics and the teaching of learning mathematics are, involve, and could be.

With the conclusion of my presentation and analysis of data related to how people view and think about what mathematics is (the philosophies of mathematics), I now return to the next area of interest that arose for me out of my telling and analysis of my story – how it is believed that mathematics should be taught and learned. In particular, I next visit the math wars, through which the dichotomized views of mathematics teaching and learning are realized.

## The Math Wars

Having explored in depth the notion of what people think mathematics is, I am now going to consider the second area of interest that, through grounded theory, emerged as important to me through my analysis of the story. This time, I am focusing on what people believe about the teaching and learning of mathematics – how it should be taught and why. For decades now, North America (and elsewhere) have been officially embroiled in the so-called "math wars," which are the consequence of different thinking about the teaching and learning of mathematics. Thus, I present as my next set of data (to be analyzed after) an examination of where the math wars originated and how they relate to the teaching and learning of mathematics.

The great variety of philosophies of mathematics previously discussed (as well as embodied mathematics and Mathematical enculturation), each valuing different kinds of mathematical knowledge and ways of knowing, clearly indicate that mathematics has never been a singular or universal entity in society (or between societies), although aspects of it have been proclaimed to be so. There have always been disagreements about what mathematics is, why we have mathematics, and how mathematics should be done and taught, and periodically different factions of thinking about these ideas would set forth their arguments against what others said and believed.

Although there was likely banter, back and forth, between the two (or more) parties involved, everyone was basically able to continue with their own work and thinking. Then, in the 1980s, all of that changed when two disparate, yet ultimately inter-influential events occurred: the release of the *Second International Mathematics Study: Summary Report for the United States* (Crosswhite, 1985), hereafter referred to as SIMS, and the National Council of Teachers of Mathematics' (NCTM) publication of *An Agenda for Action* (NCTM, 1980) and *Curriculum and Evaluation Standards for School Mathematics* (NCTM, 1989). Despite the best intentions of the new curricula implemented post-Sputnik, the report on U.S. student performance on the SIMS was dismal. Meanwhile, the NCTMs two documents in that same decade proposed a major change in how mathematics should be learned and taught, rather than proposing new content. What the NCTM recommended was that teaching and learning be "grounded in assumptions about learning being an active process rather than one of memorization and practice" (Schoenfeld, 2004, p. 266). Such a proposal was shocking to many, having been fully immersed in an educational system modeled upon the ideals of the industrial revolution and the assembly line, where mathematical content had been broken down into small, isolated chunks, the teaching of which emphasized facts and procedures.

In seeking to radically improve student performance on SIMS, and thereby help to prevent another rendition of Sputnik-like embarrassment, the California Department of Education used the NCTMs documents as a foundation for their 1992 renewal of the mathematics curricula: the *Mathematics Framework for California Public Schools, Kindergarten through Grade 12*. As one of the three states with a large enough student population to attract the attention of mainstream publishers in the U.S., California's new framework resulted in the publication of a number of (radically) new textbooks for mathematics classrooms, and the *reform* approach (emphasizing active learning of mathematics) to the teaching and learning of mathematics was officially launched. It would not be until the late 1990s that there would be students from California who had experienced the reform curricula throughout all of kindergarten to Grade 8 (and later for up to grade 12 and into post-secondary institutions). Consequently, no large-scale data collection or analysis regarding the success or failure of the reforms that were implemented would be available until at least the start of the 21st century. However, by 1996, public outcries, websites devoted to a return to the pre-Sputnik "back to the basics" approach to mathematics teaching and learning, and the support of conservative politicians, had led to the formation of a committee to write a new framework. Further, an assembly bill was also passed that made it law for the State Board of Education "to ensure that the basic instruction materials it adopts for reading and mathematics in grades 1 to 8, inclusive, are based on the fundamental skills required by these subjects" (Schoenfeld, 2004, pp. 273–274). Thus, there was a call for a new framework and a new set of resources in K-12 mathematics in California.

When the new Framework was given to the State Board of Education, it was determined that the document did not meet the new legal criteria, and the task of yet again re-writing the Framework, for a third time in the decade, was given to four Stanford University mathematics professors who completed their task in four weeks. This version of the Framework was approved by the State Board of Education of California in December, 1997.

This time, the critiquing of what a group believed about mathematics, and more specifically the teaching and learning of mathematics, was not a critique – it was a judgment. This time, the criticisms were not directed at the other party, leaving the criticized to deal with them as they would, rather they were made in a very public and political way. This time, one group had denied the second party the right to practice and think about mathematics as they chose fit. Of course, it could be argued that the first curricular changes were also critiques; however, the third rewrite of the Framework was far more than a critique. It was a condemnation of what had been implemented before it had even had a chance to be tried out, and a denial, without evidence, of the prior research. It was this assumption of authority of one group over the other, without consultation or consideration that ultimately became the starting point of the math wars, with its battles and casualties being documented and reflected upon as the decades inched forward (for example, see Becker & Jacob, 1998; Davison & Mitchell, 2008; O'Brien, 1999; Restivo & Sloan, 2007; Schoenfeld, 2004; Stemhagen, 2007). Although mathematics and mathematics teaching and learning had never been a domain of total agreement, with different philosophical and social stances leading to different ideals (Davison & Mitchell, 2008; Schoenfeld, 2004; Stemhagen, 2007), the math wars have been driven by overt political manipulation and the overabundance of negative campaigning within public media (Schoenfeld, 2004), more than ever seen before in history.

Boaler (2015) documents one of the most egregious examples of the misinformation, misrepresentation, and misunderstandings that riddle and perpetuate the math wars. It is the case of a teacher that Boaler calls Emily Moskam (a pseudonym introduced by Boaler), who had won awards for her mathematics teaching, had impressed even professors of mathematics with what here students were doing mathematically in her classroom, and whose students loved and excelled at mathematics because of how it was being taught to them. At a parents only meeting, a small group of mothers provided "data that had been fed to them, telling them that if their children continued with the new math program, then they would not be eligible to go to college, and that test scores would fall" (The Night of Nonsense section, para. 4). Test score declines was shown through the misrepresentation of data, and the eligibility for entry into different colleges was based upon the institutions answers to the question: "'Would you accept a student who had not taken any math in high school but had just talked about math'" (The Night of Nonsense section, para. 4)? Boaler also notes that

> These tactics may sound incredible, but the people involved felt justified in trying to promote their position, using any method that they could, as they believed that they were involved in a 'great educational war' and that any tactics, no matter how underhanded, are admissible when at war. (The Night of Nonsense section, para. 4)

Despite another group of parents in-depth seeking of clarification, and the confirmation that "facts" they had been presented had been misinformed by the interests of outside parties, "the damage had already been done" (The Night of Nonsense section, para. 6). Emily's school decided that all of its teachers would go back to teaching in a traditional manner, and Emily left the field of teaching completely.

As the math wars continue to have major impacts upon the teaching and learning of mathematics, it seems appropriate to better define the two major parties and their stances involved. The traditional camp, as the name suggests, is fighting for a permanent return to the status quo for the teaching and learning of mathematics, that is, a focus on traditional content, facts, procedures, algorithms, and steps. They endorse pedagogical practices that include rote practice, rote memorization of rules, teaching by telling, and relying on an outside authority such as the teacher, textbook, or answer key (Schoenfeld, 2004). For traditionalists, learning mathematics is only about obtaining the content, and that content is precise and deliberate. Efficiency (in the form of speed) in doing mathematics is of the upmost importance and thus students need to only learn the best way to get the correct answer.

On the other side, the reform camp, building off the NCTM's (1989) standards document's descriptions of what mathematics teaching and learning should be like, is more focused, at first, on the processes of thinking, doing, and learning mathematics, rather than on any particular content to be learned. Thus, the reformists believe that mathematics should be learned through student investigation of, or inquiry into, substantial problems. Moreover, they argue:

> the classroom teacher should act as a stimulant, sounding board, and guide in student problem solving; that students should be encouraged to discuss mathematical ideas and discoveries with classmates and with the teacher; that the classroom activity should include frequent challenges to students to develop justifications for their ideas and discoveries; and that students should be encouraged to use calculators and computers in their mathematical expressions. (Schoen, Fox, Hirsch, & Cox, 1999, p. 446)

Reasoning and problem solving are the most highly valued aspects of mathematics teaching and learning within the reform camp.

As noted earlier, there are more than just the fighting factions involved. It is also of great importance to acknowledge and consider the plight of the students, teachers, and parents (such as Emily, her students, and their parents) who are caught in the middle of this dichotomy of thinking about mathematics teaching and learning. They have little to no influence on what is being done, yet have to persevere it being done, regardless of the impacts and consequences. In this regard, a number of researchers, scholars, and authors have suggested the need to find a middle ground, and in some cases have hypothesized about how to attain it (Loewenberg Ball, Ferrini-Mundy, Kilpatrick, Migram, Schmid, & Scharr, 2005; Reys, 2001; Wallis, 2006). These common grounds often involve specific attention to learning algorithms through pedagogical strategies intended to develop the students' understanding of the algorithms. To date, there is no substantial evidence (or even acceptance of the little evidence that does exist) of either the effective implementation of such a common ground or the implications of such on student learning.

And so, mathematics teaching and learning remains stuck in the battlefield of the math wars, often simmering in the background, occasionally brought to the forefront of everyone's thinking through blasts of often less than professional claims. Arguments related to educators not knowing enough about mathematics are countered with arguments about mathematicians not knowing enough about education; parents are caught (with their children) bouncing back and forth between mathematics texts that they can recognize and others that they can not. Most recently, Western Canada has become the newest front line to emerge as curricula renewal has brought to the forefront these same controversies within the teaching and learning of mathematics (for example, see CBCNEWS, April 5, 2007; Murphy, April 15, 2012; http://wisemath.org/). Twenty years on, resolution remains as elusive as ever, suggesting that a new approach, a new way of understanding the problem, needs to be sought.

**Analysis of the Math Wars**

Based on the above understandings of the math wars, I will next proceed to my analysis of them. This analysis will again first discuss the prominent features of the math wars, followed by a discussion of the responses of the Traditional Western worldview and an Indigenous worldview to the wars, and then will end with the coding and discussion of concepts that emerge from the above explanation and analyses of the math wars.

**Prominent features in the math wars.**

It was a double spark that ignited the math wars: failure of the post-Sputnik curriculum reforms to result in the desired improvement in students within mathematics, and the NCTMs arguments for mathematics learning being a dynamic rather than passive enterprise. With the failure of the most previous curriculum reforms, these two factors ultimately divided people's thinking about the teaching and learning of mathematics into those who wanted to return to the status quo of the pre-Sputnik era, and those who wanted to embrace the calls for active learning from the NCTM.

Since that time, the math wars have continued to bounce (almost always without consent or agreement) students, teachers, parents, and administrators between the two opposing camps of those supporting the traditional (pre-Sputnik) and the reform (NCTM based) approaches. The wars have over time flooded media with information and misinformation, and they have often been politically and popularly (through mathematical "celebrities") supported and rebuked.

In most general terms, the traditional approach to the teaching and learning of mathematics as argued for within the math wars emphasizes rote practice, rule memorization, practice of teacher-demonstrated skills, and reliance upon a source of authority (the teacher, textbook, or answer key) through which mathematical knowledge must be obtained. Within the traditional approach, individual aspects of mathematical knowledge are presented directly to students who then memorize and practice using that knowledge in familiar contexts to build up efficiency, particularly in the form of speed.

Alternatively, the reform approach embraces pedagogical practices that emphasize the construction of knowledge that is understood as well as known. These approaches, such as inquiry and problem-centered learning, aim to engage students in creating mathematical knowledge that they can use to solve problems and answer questions that have piqued their interest. Thus, the reform camp within the math wars seeks to engage students in mathematical thinking and doing in order for them to learn particular mathematical content. Reform-based classrooms encourage dialogue, debate, conjecture, and experimentation.

The math wars are thus in many ways a dichotomized view of the teaching and learning of mathematics, the traditional approach sitting at one end of the spectrum, while the reform approaches sit at another. The dichotomy is not absolute however, because both camps desire for students to learn particular mathematical knowledge. It is only in how each camp envisions students achieving this goal that not only distinguishes them, but also puts them in conflict with one another. With these understandings of the math wars and its two factions, I now turn to the results of hermeneutically considering how the Traditional Western worldview would respond to the prominent features of the math wars.

**Dialogue with the Traditional Western worldview.**

As is central to its worldview positioning, the Traditional Western worldview would look for the "right" way to teach and learn mathematics within the positioning of the two sides within the math wars. Another significant feature of the Traditional Western worldview when reflecting on the math wars from that perspective is its emphasis on knowledge of value being known by, and shared through, reliable sources with a specialized background. Based upon these two points, the post-Sputnik curriculum reform failure would be viewed as a failure to call upon the correct specialists and specialized knowledge related to mathematics and the teaching and learning of mathematics, which consequently resulted in the "right" way of teaching and learning not having been incorporated. However, a person grounded within the Traditional Western worldview would believe that such specialists can be called upon, and that it is most likely that the past "right" way (that is, pre post-Sputnik) had been wrongfully dismissed as is held by the traditionalist camp.

The argument of the reform camp that mathematics should be actively learned through the initiatives and understandings of students would be unequivocally rejected within the Traditional Western worldview. Students are not specialists, therefore it would be the view of a person grounded within the Traditional Western worldview that students could not be expected to develop mathematics themselves.

The fact that the math wars continue to rage on, and that teachers, students, parents, and administrators often are the victims of these wars, would also be squarely placed on the failure to make the "right" choice and to return to mathematics teaching and learning which is authority and specialist centered rather than student and understanding centered. Further, a person grounded within the Traditional Western worldview would view political and popular arguments for a return to the traditional approaches to the teaching and learning of mathematics as rational and sound.

For all of the above reasons then, a person grounded within the Traditional Western worldview would find themselves in support of the traditional camp within the math wars, praising the authority, rational basis, and emphasis on specialization and compartmentalization of the knowledge that it seeks and incorporates. Alternatively, the same person would reject the reform camp's position on the basis of its incorrect assumption that students would be able to create significant, specialized and compartmentalized mathematics by engaging in problem solving and inquiry. Instead, they would argue that problem solving and inquiry, if absolutely desired, would necessarily have to follow students first being taught the "right" mathematics and doing of mathematics by a knowledgeable person or authoritative text.

Overall then, the Traditional Western worldview would not find any valid reason for the continuation of the math wars. For a person grounded within this worldview, there is a universally correct side to these battles, that of the traditional camp. With this understanding from the perspective of the Traditional Western worldview, I next turn to a discussion of how a person grounded within an Indigenous worldview would respond to the math wars.

**Dialogue with an Indigenous worldview.**

As has become a common occurrence, the math wars appear very different from the perspective of an Indigenous worldview than they do through the lens of the Traditional Western worldview. For example, a person grounded within an Indigenous worldview would find value in both of the triggers for the math wars; however, they would also find that the isolation of these two factors, and the failure to consider other issues, such as the struggles of many students (including Indigenous) with mathematics in general, a limited mindset to have when considering the teaching and learning of mathematics.

Further, from the perspective of an Indigenous worldview, the flip-flopping between the two camps would seem unnecessary and therefore irresponsible, as would the political and popular media activities that promote these abrupt responses and changes. From the perspective of an Indigenous worldview, depending upon the context in which the knowledge being sought is needed, both approaches to the teaching and learning of mathematics could be appropriate. Instead, the exclusion of consideration of the context driving the two different approaches would be seen to be the real cause for the disagreements. Thus, a person grounded within an Indigenous worldview would not accept the necessity of dichotomizing the two approaches, rather they would find a place for both approaches, while keeping other space open for combinations of both and for approaches not yet even considered.

Thus, although a person grounded within an Indigenous worldview would be pleased to see the openness in terms of the ways of knowing valued within the reform approaches versus that of the traditional approach, this would not be enough for that person to be situated within the reform camp. Neither would the traditional camp offer a suitable place for that person's views on the teaching and learning of mathematics to reside. Instead, a person grounded within an Indigenous worldview would move through, between, and even beyond the two camps, choosing where to locate based upon the context in which they are seeking to gain or share mathematical knowledge.

With these understandings of how the math wars relate (or do not relate) to the Traditional Western worldview and an Indigenous worldview, I can now turn to the final part of the analysis of the math wars. Returning to grounded theory, I now present and discuss the concepts that I have coded as being found within the data and the analysis of it so far.

**Coding and explanation.**

More so than any of the previous data that I have coded for concepts, in many ways the math wars emphasize a dichotomization of each of the concepts. Just as the two camps tend to be positioned as a dichotomous pair, through the two approaches to the teaching and learning of mathematics, the concepts previously identified tend to diverge in their explanations through the lenses of the traditional and reform camps.

The first of the concepts to consider is that of hierarchy. The traditional approach to the teaching and learning of mathematics emphasizes a hierarchy of knowledge by controlling the order in which the mathematics content is taught, and by delegating the teaching of mathematics to specialists and authoritative textbooks. The reform camp, on the other hand, encourages teaching and learning of mathematics where the content learned is dependent upon the problem solved or the inquiry carried out, both of which may come from the students rather than the teachers. This means that although the teacher may do some “fancy footwork” to ensure the inclusion of a particular piece of mathematical content, the content that is learned is most often the result of the motivations and interests of the students. Consequently, the hierarchy of knowledge, if it can be considered a hierarchy, is based upon the knowledge that students construct through their solving of problems and engaging in inquiries.

Specialization as a concept is also very different within the two camps of the math wars. For those favouring the traditional approaches to the teaching and learning of mathematics, the teacher must be a specialist in mathematical knowledge, or have access to resources, which contain the specialized mathematical knowledge to be learned. Conversely, the reform approaches to the teaching and learning of mathematics seek ways for students to develop their own specialized knowledge and ways of knowing of mathematics.

In relation to the singularity of mathematical knowledge, the traditional approaches emphasize both the “right” way of knowing and learning mathematics and the singular pieces of mathematical knowledge. The reform camp instead focuses on students finding “their own right” ways to do and know mathematics, and the mathematical knowledge that is to be gained by the students through such approaches is to be comprised of integrated pieces of mathematics that are contextually grounded.

The categorization and isolation of mathematical knowledge is likewise dichotomized between the two camps. The traditionalists hold that mathematical knowledge should be taught in rote pieces of knowledge, including facts and skills, which are categorized and kept isolated from other mathematical knowledge so that they will not be confused. The reformists, on the other hand, have students engage with the learning of mathematics in ways that integrate and relate various mathematical ideas and skills within particular contexts and investigations.

Thus, the two camps also have alternative views to relationship and context. In particular, the traditional approach to the teaching and learning of mathematics avoids the presentation of relationships or contexts, unless absolutely required for the students to acquire the mathematical knowledge being presented. Relationship and context, are dissimilarly the foundations upon which mathematical knowledge is to be constructed within the reform approaches to the teaching and learning of mathematics.

As referenced earlier, power and authority within the traditional approaches to the teaching and learning of mathematics are afforded to the teachers and textbooks, in the hopes that the students will ultimately gain the knowledge necessary to give them the more power and authority. Contrarily, within the reform approaches to the teaching and learning of mathematics, students are afforded the authority and power to create mathematics, which they can use with authority in relevant situations.

It is only in light of the concept of abstraction that the dichotomy between the two approaches to the teaching and learning of mathematics ceases to exist. This is because, ultimately, both the traditionalists and the reformists seek for students to gain abstract mathematical knowledge. One assumes that it must be given directly and specifically to the students (the traditional approach), while the other assumes that students can come to know the abstract mathematics through constructing it for themselves within meaningful contexts (the reform approach).

Not only are all of the previously identified concepts again present within the math wars data, but also, regular interactions and inter-relationships between the concepts are again present.

Thus, the saturation of the concepts, and a broader conceptual category continue to emerge through the analysis of the math wars.

With this conclusion to the analysis of how people think about the teaching and learning of mathematics, I once again return to the areas of interest that emerged from the analysis of my story for the source of my next set of data. In this case, I now turn to how mathematics relates to culture and individuals, and more specifically to Indigenous students' mathematics struggles and ethnomathematics.

**Indigenous Students in Relation to Mathematics and Ethnomathematics**

Although likely the most publicized, the math wars are not the only context in which mathematics educators are concerned about what is happening to student learning of mathematics. In 2003, the U.S. Commission on Civil Rights reported:

> As a group, Native American students are not afforded educational opportunities equal to other American students. They routinely face deteriorating school facilities, underpaid teachers, weak curricula, discriminatory treatment, and outdated learning tools. In addition, the cultural histories and practices of Native students are rarely incorporated in the learning environment. As a result, achievement gaps persist with Native American students scoring lower than any other racial/ethnic group in basic levels of reading, math, and history. Native American students are also less likely to graduate from high school and more likely to drop out in earlier grades. (p. xi)

However, this is not just a U.S. issue. A decade later, there is evidence that Saskatchewan's First Nations and Métis students (the fastest growing population set in the province) are continuing to fall well behind their non-Aboriginal counterparts in mathematics (Saskatchewan Ministry of Education 2008, 2009b, 2010).

For example, in 2006, 43.1% of the Aboriginal population who were of an age to have been expected to complete grade 12 had not graduated from high school compared to 15.1% for the same age range in the non-Aboriginal population (Saskatchewan Education, 2008). Further, there is consistentaly a documented 20% difference in marks in Grade 11 mathematics (the last grade in which mathematics is compulsory for graduation) between Aboriginal and non-Aboriginal students in the province (Saskatchewan Ministry of Education 2008, 2009b, 2010). Moreover, this is not a phenomenon of just Saskatchewan and the U.S. – it has been noted throughout all of North America (Aitken and Bruised Head 2008; Cheek 1984; Graham 1988; Mather 1997; Scott 1983; Trent and Gilman 1985), Australia (Howard & Perry 2005), and around the world (Greer, Mukhopadbyay, Powell, & Nelson-Barber, 2009; Powell & Frankenstein 1997b).

Within these different reports however, there is also evidence that the world's Indigenous peoples are more than capable of complex mathematics, often produced and used in ways foreign to Western academic mathematics. For greater understanding of this reality, one must turn to the field of ethnomathematics, which focuses on the relationships between mathematics and culture, and provides evidence of how the Mathematics (to borrow the capitalization of Bishop) taught in schools today is culturally biased. Ethnomathematicians challenge beliefs in the authority of

Western, abstract mathematics, such as in the downplaying of the Egyptian way of determining surface areas of cylinders that was overlooked in favour of the Greek formulas (Powell & Frankenstein, 1997a). Further examples of this kind of hierarchical authority structuring are found in the way solutions to non-standard problems by Africans were dismissed as "flukes" (Lumpkin, 1997), and how the knitting of socks was classified as women's work rather than mathematics, despite both involving the solving of the same mathematical problem (Harris, 1997).

Ethnomathematical research has not only brought to light this assumed hierarchy in kinds of mathematical knowledge. It also provides evidence of overt misunderstanding and assumptions about ways of mathematical thinking and knowing. For example, Ascher and Ascher (1997) share an often-recounted story of researchers who offered to buy a sheep from an Indigenous sheepherder. When informed that the price would be two sticks of tobacco, the researchers left to get the payment. Upon their return, the researchers offered the sheepherder four sticks of tobacco in payment for the original sheep and one other. The researchers reported that: "the herder agrees to accept two sticks of tobacco for one sheep but [became] confused when given four sticks of tobacco after a second sheep is selected" (p. 29). From the researcher's perspective this was conclusive evidence that "that the herder cannot comprehend the simple arithmetic fact that 2 + 2 (or 2 x 2) = 4" (p. 29). Ascher and Ascher point out that it was not that the sheepherder was not able to think mathematically that this misunderstanding occurred; rather, it was because the researchers had failed to take into account the context, that "Sheep are not a standardized unit" (p. 29).

Similar to the previous sheepherder example, another frequently situation discussed within ethnomathematical papers and conversations is that of research done in another Indigenous community designed to gauge the level of mathematical knowledge within the community using a task modeled off of one of Piaget's sorting activity. In the task, twenty different objects that could be classified as being food, clothing, tools, or cooking utensils, were placed in front of the research participants. Each participant was then asked to sort the objects. The researchers were dismayed to note that not only were the participants not constructing the obvious categories, but rather putting of objects together (such as an orange with a knife), but they often did so with the explanation, "'that a wise man would do things in the way this was done'" (Powell & Frankenstein, 1997c, p. 197). Frustrated, one of the researchers eventually questioned a participants "'how would a fool do it?'" (Powell & Frankenstein, 1997c, p. 197), to which the participant responded by sorting the objects into the five categories that the researchers had been expecting. In both the case of the sheepherder and the sorting task, the researchers had different ideas about how a given problem might be solved than the research participants had, and as a result, at least by the researchers and for a period of time after, the participants were deemed to have been inferior in their mathematical thinking.

These examples serve to demonstrate that it might not be being able to do mathematics that eludes Indigenous students; rather, it may be what schools assume that doing mathematics looks like is the issue. Regardless of the reason, the failure of many Indigenous students to succeed in the learning of mathematics is of grave concern, for as Graham (1988) notes, "without Western mathematics, Aboriginal children are denied access to further education and to the knowledge and power inherent in the social institutions which, even today, influence the way Aboriginal people live their lives" (p. 120). In other words, the continuation of this negative relationship between Indigenous students and the learning of mathematics is contributing to the continuation of oppression, hegemony, and colonization.

The combination of ethnomathematical research and the documented academic struggles of many Indigenous students with mathematics, since at least the 1990s, has resulted in researchers, educators, and curriculum developers struggling with the question of how to make mathematics more accessible and meaningful for Indigenous students. Around the world, various strategies for improving the academic achievement of Indigenous students in Western mathematics have been (and continue to be) developed, implemented, and researched.

In New Zealand (Barton, Fairhall, & Trinick, 1998; Barton 2009), researchers and Indigenous peoples have been working on the creation of a mathematics register (which we define as the language, both vocabulary and grammar, used within Western mathematics) within the Mäori language. Barton et. al. (1998) note that this work has resulted in "The creation of a Mäori mathematical discourse [that] has moved the language towards English modes and conversations" (p. 7) . Unintentionally, these endeavours to bring school mathematics into the Indigenous languages have also resulted in further losses to the languages and culture.

Some researchers have also sought to find other ways to connect Indigenous students to their own mathematics and culture. For example, Lunney Borden and Wagner (2007), have invited Indigenous students to bring into their school mathematics classrooms the mathematics from their homes and community. Others, such as Neel and Fettes (2010) have incorporated Aboriginal images, art, artifacts, and symbols into their school study of mathematics.

Alternatively, researchers such as Sternberg and McDonnell (2010) have worked on ways to embrace traditional Aboriginal teaching and learning strategies, such as an emphasis on learning from place, in order to engage Aboriginal students in the learning of Western mathematics. In terms of these approaches to connecting with Indigenous students, researchers and mathematics educators need to proceed with caution. It is important that when trying to help Indigenous students connect with school mathematics that the cultural contexts, ideas, artifacts, and places do not become merely objects of mathematical thinking and knowledge, abstracted from their culture and contexts (three Saskatchewan Cree Elders, Personal Communication, 2006).

Regardless of the approaches taken, the achievement gap for Indigenous students in school mathematics continues for many Indigenous, and non-Indigenous students. What is also important to note is both ethnomathematics and the struggles of many Indigenous students with learning mathematics may very well be related to each other, and that "different cultures can produce different mathematics, and the mathematics for one culture can change over time, reflecting changes in the culture" (Powell, & Frankenstein, 1997a, p. 6).

**Analysis of Indigenous Students in Relation to Mathematics and Ethnomathematics**

Based upon the above understandings of Indigenous students in relation to mathematics and ethnomathematics, I will next proceed to my analysis of these two interrelated sets of data. This analysis will again first discuss the prominent features of the relationships between Indigenous students and mathematics as well as of ethnomathematics, followed by a discussion of how the Traditional Western worldview and an Indigenous worldview would respond to these two fields of knowledge, and then will end with the coding and discussion of concepts that emerge from the above explanation and the analyses that follow.

**Prominent features of Indigenous students in relation to mathematics and mathematics.**

The above discussion of what is said in the literature regarding how (many) Indigenous students struggle with the learning of mathematics demonstrates that these struggles are not just confined to within the boarders of Saskatchewan or even Canada, but can be found within Indigenous populations around the world. Despite their struggles in school mathematics, however, ethnomathematical research has demonstrated repetitively that all people, including

Indigenous peoples, are capable of complex mathematical thinking and doing.

Ethnomathematics also reveals how undervalued and misunderstood Indigenous mathematics (and all mathematics that was not made part of the Western mathematical knowledge system) is. In some cases, Indigenous mathematics was usurped by Western mathematics, being given abstract symbolic representation, and then claimed as being Western in origin. In other cases, Western ignoring of context lead to incorrect conclusions about the mathematical ability of Indigenous peoples.

Likewise, equivalent mathematical thinking that was represented and carried out in ways different from the formulaic processes of Western mathematics was not even considered mathematics. Further, assumptions about the purpose and hierarchy of different types of knowledge led many mathematical and anthropological researchers to believe that different Indigenous groups were mathematically incompetent, while it was merely the valuing of different assumptions and kinds of knowledge resulted in disparity between the mathematics of the researcher and the mathematics of the researched.

In general, ethnomathematics research sheds light upon differences between cultural perceptions, representations, and uses of mathematics, particularly between Western mathematics and Indigenous mathematics. The great importance and value placed on mathematical knowledge within Western society thus raises concerns about how Western mathematics presents mathematical thinking and mathematical knowledge, and what ways of knowing and doing mathematics that it values or disadvantages.

Consequently, many mathematics education researchers have engaged in various approaches to finding ways to connect Indigenous students to mathematics and mathematics to the Indigenous students. Some of these attempts have resulted in more positive attitudes of Indigenous students to the teaching and learning of mathematics, while others have caused unwanted changes to Indigenous languages. Even with the variety of attempts made to change the teaching and learning of mathematics for struggling Indigenous students, and despite indications of some improvement in their academic progress, the achievement gap in mathematics continues for many Indigenous students.

Thus, it is important to recognize that there is a potential for a strong relationship between knowledge and understandings emerging from ethnomathematical research, the struggles of many Indigenous students to have success in school mathematics, and the various approaches taken by researchers to try to bridge Indigenous students' knowledge and Western mathematical knowledge. With these understandings of these three areas and their relationships, I now turn to the results of hermeneutically considering how the Traditional worldview would respond to the data just presented.

**Dialogue with the Traditional Western worldview.**

From the perspective of the Traditional Western worldview, the worldwide struggle of many Indigenous students in regards to the learning of Western mathematics would necessarily be seen as being related to issues of the students, and not of the mathematics or how it is being taught and learned. This response would be deemed appropriate within the Traditional Western worldview because it would be argued that no other culture has this nearly en masse issue with achieving in mathematics (a statement that could be countered by considering current achievement levels of different nations on international mathematics testing, but will not be pursued further within this dissertation); therefore, what is being taught and how it is being taught must be the "right" mathematics and the "right" way of teaching it. In the same vein, the Traditional Western worldview would not accept the alternative forms of mathematical knowledge that the ethnomathematical research highlights since there must be only one "right" way of knowing and doing mathematics. Everything else must necessarily be classified differently or assumed false.

For these reasons, mathematics education researchers seeking alternative teaching and learning approaches for Indigenous students would be deemed to be engaging in unnecessary research from the perspective of the Traditional Western worldview. Moreover, engagement of Indigenous students, as well as possibly non-Indigenous students in such learning pursuits would be viewed as impractical, and a waste of time. A person grounded within the Traditional Western worldview would substantiate such claims by pointing to the unintentional damage that some of these approaches have had on Indigenous languages, and the failure of these approaches to completely (or even significantly) influence the achievement gap.

Overall then, from the perspective of the Traditional Western worldview, the seeking of reasons for the struggles of the Indigenous students within mathematics and the teaching and learning of mathematics would be viewed as an inconsequential and insignificant pursuit. Likewise, focusing on particular, context-based examples of ethnomathematical thinking that do not fit within the parameters of Western mathematics would also be of little value to a person grounded within the Traditional Western worldview. With these understandings of how the Traditional Western worldview aligns (or in this case, does not align) with the data under consideration, I now discuss how a person grounded within an Indigenous worldview would respond to this same information.

**Dialogue with an Indigenous worldview.**

From the perspective of an Indigenous worldview, the achievement gap in mathematics for many Indigenous students around the world would be viewed as a context for concern, not because the students are Indigenous, but because an Indigenous worldview would view the situation as problematic for the good of all. Likewise, a person grounded within an Indigenous worldview would want to know about the downplaying and misunderstanding of any knowledges, as this worldview seeks and values, rather than denying, diversity in both kinds of knowledge and ways of knowing. Further, a person grounded within an Indigenous worldview, while questioning what good comes from the appropriation of the knowledge of others, would be greatly concerned with the insistence of Western mathematics to take contextualized knowledge and abstractly represent and categorize it.

An Indigenous worldview would also insist upon the valuing of all mathematical knowledge, whether its representation or use conformed to the norms of Western mathematics (abstract and rational), and regardless of how the knowledge has come to be known. Culturally centered and derived mathematical knowledge outside of Western mathematics would be viewed as being as valuable as that within Western mathematics. These stances upon what knowledge and ways of knowing should be valued in mathematics, and the teaching and learning of mathematics would be further fortified within an Indigenous worldview when the role that mathematics knowledge plays within Western society is added to the picture. Thus, the attempts to find different ways to approach the teaching and learning of mathematics by researchers would be applauded by a person grounded within an Indigenous worldview. Such research would be viewed as research that is being done for good reasons as it is the seeking of knowledge that is for the good of all. Even with the mistakes made, and the limitations on progress observed, such attempts would be celebrated and future attempts would be supported.

With these understandings of how the relationships between many Indigenous students (and cultures) and mathematics, through both ethnomathematics research and the work of mathematics education researchers, I can now turn to the final part of the analysis of this section of data.  Returning to grounded theory, I now present and discuss the concepts that I have coded as being related to the data shared and the analysis of it so far.

**Coding and explanations.**

As has become the norm, this section of my data and its analysis again brings to light the same concepts that have been previously noted.  In particular, much of the coding of the concepts comes from the worldview responses to the struggles of many Indigenous students with mathematics, ethnomathematics, and the approaches that have been developed and tried by mathematics education researchers.

As noted previously, hierarchy and specialization are again present within the response of the Traditional Western worldview through its justification of the devaluing of non-Western mathematics.  Hierarchy and specialization are apparent in thinking that the cause of many Indigenous students struggles with mathematics and the learning of mathematics must be the result of their failure to do mathematics the “right” way, rather than it being a consequence of how mathematics has been presented and taught.  Hierarchy and specialization also play a role in the ethnomathematical examples that were shared.  Ethnomathematical knowledge that was contextually based, or that considered a different interpretation of what kinds of knowledge were of value was deemed by prior researchers to be indications of mathematical inadequacy rather than representational difference.  In this way, the people who demonstrated different kinds of mathematical knowledge and ways of knowing from the researchers (who assumed they were specialists at a high level within the hierarchy of knowledge) were deemed inferior and lacking in knowledge of value.

The failure of prior researchers to acknowledge the diverse ways of knowing and kinds of knowledge that are present in their recounting of their research also highlights their emphasis on singularity and abstraction of mathematical knowledge.  Conversely, the mathematical knowledge that they rejected challenges the assumption that singularity and abstraction is necessary in order for knowledge to be valuable mathematical knowledge.

Emphasis on the categorization and isolation, as well as the abstraction of mathematical knowledge, can be seen in the manner in which the mathematical knowledge and ways of knowing were researched prior to (and unfortunately at times still) the emergence of ethnomathematics.  The researchers went in with specific mathematical knowledges that they were wanting to research (such as sorting of objects) and with very specific responses to their research questions being expected (such as by type rather than by relationships). Contextual factors were ignored in their analysis of the data that was collected (in order to provide objectivity). The consequence was that in many such cases of research, the categorization of mathematical knowledge held by the researcher did not align with that of the researched, and the researchers attempts to isolate and abstract the mathematical knowledge away from the context created a greater divide between the Indigenous knowledge and the researcher's interpretation of it.

As was just noted, context played a huge role in the differences between how the Indigenous people were working with and thinking about mathematics; while the original researchers ignored the context. Ethnomathematical researchers revisited this original research, highlighting the significance and role of context. Context also plays an important role within the various approaches to teaching mathematics to Indigenous (and other) students that have recently been, and continue to be, researched.

A dichotomy in perceptions regarding the relationship between knowledge and knower is thus challenged through this section of my data. Whereas the original researchers assumed that knowledge and knower are separate from each other, ethnomathematics and the new approaches to the teaching and learning of mathematics seek to identify, accommodate, and encourage relationships between the knowledge and the knower. They do so through the ways of knowing allowed and through the contexts within which the knowledge is presented, gained, and housed.

Finally, the concept of power and authority is also present within the discussion of the struggles of many Indigenous students with mathematics as well as within ethnomathematics (and the research findings that it questions), as well as approaches to the teaching and learning of mathematics. Currently, mathematical achievement is exerting tremendous power and authority over many Indigenous (and other) students, denying their ability to pursue many options within their futures. Ethnomathematics, on the other hand, is challenging the assumed power and authority of Western mathematics and Western mathematical ways of thinking by bringing to light misinterpretations and otherwise ignored mathematical knowledge and ways of knowing. Further, to this challenge of the power and authority of Western mathematics, the new approaches to the teaching and learning of mathematics for Indigenous (and other) students likewise confront the restrictive practices of mathematics teaching that denied students to engage with mathematics in meaningful, relational, and contextual ways.

As can be seen within the above discussion of the conceptual codes, the concepts again are becoming more saturated, with only more clarity about the horizon of understanding of the particular codes emerging.  Likewise, the merging and inter-relating of these concepts continues to suggest a broader conceptual category is developing.

With this conclusion to the analysis of the data speaking to the relationship between cultures (and individuals) and mathematical knowledge and ways of knowing, I once again return to the areas of interest that emerged from the analysis of my story for the source of my next set of data.  In this case, I now consider how curriculum represents mathematics content with an eye to emerging mathematics content.  In particular, I have chosen to consider the newly emerging mathematical content area of risk, risk analysis, risk-based decision-making, and risk management.

## Risk and Risk Education

The final area of interest that emerged from the grounded theory analysis of my first data set, my story, relates to how mathematics knowledge and ways of knowing are present and represented within curriculum. As this is a very broad topic, I have chosen to focus on one particular area of mathematics content that has only recently begun to appear within mathematics curricula documents, that of risk education. This choice was deliberate on my part, as I am hoping that by presenting and analyzing a mathematics topic that is in its curricular infancy, my analysis may be able to better identify different possibilities for curricular implementation of risk topics and knowledges.

Risk, risk analysis, risk management, and risk-based decision-making are ubiquitous ideas and terms in the modern world; however, what is meant by risk and how one can analyze, manage, or make decisions based upon risk remains if not contested, then inconclusive. Borovcnik and Kapadia (2011), define risk as "a situation with inherent uncertainty about the (future) outcomes, which are related to impact (cost, damage, benefit)" (p. 5503). Within much of risk research, there is general agreement that "there are two levels of criteria for making decisions: personally preferred ones and rationally bound ones" (p. 5503). There even is agreement as to the origins of the knowledge held in relation to each of the criteria (affective/emotional responses and scientific methods, respectively); however, the perceived worth of each of these criteria by researchers is not so clear cut. Further, as with other emerging and prominent features of the modern world, like technology, there is a proclaimed need for the study of risk to be part of students' educational experiences. Hence, there is a need to consider risk education –what should it look like, and how can it happen, and educational researchers have begun to investigate these questions.

### Risk Education

Since risk impacts our lives in so many ways, it is not surprising that many educational systems and researchers are looking for ways to embed risk assessment and management and risk-based decision-making into the K-12 school system. As an example of how and the extent to which risk education is being incorporated into school curricula, an examination of examples from Saskatchewan's mathematics curricula are now presented.

#### Risk education in k-12 Saskatchewan mathematics curricula in its infancy.

The first example is an outcome in the Grade 12 Workplace and Apprenticeship

Mathematics curriculum (Saskatchewan Ministry of Education, 2013b): "Analyze and interpret problems that involve probability", and more specifically, indicator 1.6 for this outcome: "Explain, using examples, how decisions may be based on a combination of theoretical probability calculations, experimental results and subjective judgments" (p. 26). Such decisions may well, in fact often likely would, be in relation to situations involving risk. A similar outcome in the course Grade 12 Foundations of Mathematics (Saskatchewan Ministry of Education, 2013a), " Interpret and assess the validity of odds and probability statements" has the indicator "1.5 Explain, using examples, how decisions may be based on probability or odds and on subjective judgement" (p. 71), which is also easily connected to risk assessment and management, as well as risk-based decision-making. Thus, within these two courses, students should be engaging in risk analysis and risk-based decision-making through the consideration of both objective and subjective knowledge. The two indicators give some direction towards what students might learn with respect to thinking about risk, namely probability, odds, and subjective judgments in decision-making; however, what kinds of subjective knowledge to be considered and are to be valued in these outcomes remains undefined.

**Risk education research.**

Beyond what the limited references to risk, risk analysis, risk-based decision making, and risk education can be found within curriculum documents, there have been two major studies done to investigate how to best teach students about risk. The first of these studies is Martignon and Krauss' (2009) research, which aimed to develop within students thinking a "chain of competencies that make up good decision making for informed consent in basic domains of modern life like those of medical and investment decisions (P. 229). Working with grade four students, used hands-on activities and tools designed to strengthen student learning of these competencies and looked for ways to incorporate these strategies into pedagogical design. In the activities, the students moved from investigating how to make logical inferences within the context of if-then statements, to investigating the inclusion of conditional probabilities, to the comparing of proportions. Martignon and Krauss explain that "[students] need to understand conditional probabilities for determining the validities of features and they need to make comparisons between different validities of features for establishing rankings among features… These competencies are at the core of risk assessment" (p. 231), and "the comparison of proportions is essential in comparing feature validities and for assessing risks" (p. 232). Overall, the researchers describe the ordering of their activities as being based upon a "a 'historic trajectory… from logic to probability" (p. 238), giving the students a historically accelerated experiential learning of decision-making and reasoning with risk.

The second major study into how to teach about risk is that of Kapadia, Kent, Levison, Pratt, and Yogui, explain that "going beyond the idea of risk in statistical theory, we are trying to understand how personal values and models influence thinking about risk and the process of decision-making, and the implications of this for classroom practice" (Kent, Pratt, Levinson, Yogui, Kapadia, 2010, p. 1). Thus, different from Martignon and Krauss' (2009) rational focus on the historical trajectory from logic to probability, this second group of researchers were instead focusing on how affective (emotional) knowledge impacted risk analysis and decision-making, thereby assuming "that decision-making involves the coordination of different kinds of information, based on quantitative models and personal value systems and judgements" (p. 1). This study was also different from the first in that the research participants were mathematics and sciences teachers who worked in pairs (one from each specialty). This decision was based upon a second assumption – that science teachers would be more familiar with the socio-scientific aspects (including affective ones) of understanding and working with risk, while the mathematics teachers would be more familiar with the stochastic aspects of understanding and working with risk. Overall, the researchers hoped that through such groupings the participants would come to a better understanding of risk and decision-making processes.

The choice by these researchers to involve teachers, rather than students, was made in the hopes that the dynamic software tool (Deborah's Dilemma) that the teachers would engage with might later inform their teaching decisions, or even be incorporated into their classrooms. This program asked the participants to make decisions on Deborah's behalf regarding undergoing surgery for a chronic medical condition. The participants were provided a variety of statistical and affective information regarding Deborah's case upon which to reach their decision. It is important to note that the information provided was often lacking pertinent information, and in the case of the affective data there was no pre-assigned measure of importance to Deborah or the decision. After recognizing that the participants were initially treating the two sets of data as disconnected information. To help the participants consider alternate possibilities for the relationship between the data sets, the researchers added a component to the software called the 'Risk Mapping' Tool. In explaining the tool, Kent et.al. (2010) explained that,

> Whilst the mapping tool does enforce the association of impact and likelihood with each hazard, we did not enforce any model for how these relate to 'level of risk'. It was exactly at this point where we hoped users would express their personal models for the situation, providing us with a window on their thinking about risk, (Kent, et. al., 2010, p. 4)

once again assuring the possibility for both rational and affective reasoning to emerge. With this third tool, the participants were found to be able to better coordinate their thinking about the hazards and about the impact – that is the rational statistics and the affective impingements.

Since what risk education might, let alone should, include is still an open discussion, it is beneficial to next provide two examples related to risk analysis and risk-based decision making that are based upon neither rational statistical evidence, nor affective or emotional impacts. Both of these examples demonstrate another way of knowing about risk and making decisions and are based within and around Indigenous communities and their traditional knowledges.

**Navajo Plague, 1993**

In the spring of 1993, a healthy, newly engaged Navajo woman of 24 became sick with "a stuffy nose, a dry cough, aches, and little else. It looked like an ordinary case of the flu" (Arviso, & Cohen, 1999, p. 117). The following day, the woman "showed up in Crownpoint in severe respiratory distress and hypoxic ... She'd died a few hours later" (p. 118). On the day of her funeral, her 19 year old fiancé became similarly ill, was "brought to the GIMC emergency room in full respiratory and cardiac arrest and died shortly thereafter" (p. 120). These were the first two patients of a soon to be epidemic that was spreading through the Four Corners – the name given to a region within Arizona, New Mexico, Colorado, and Utah – in which a number of Navajo reservations are located. Although the as yet unidentified disease was seemingly targeting only Navajo people (hence the name 'Navajo Plague'), restaurants and businesses in communities adjoining the reservations began refusing to serve anyone who appeared to be Navajo in descent, people began cancelling vacation reservations in the south, and "the national media jumped to the conclusion that it was *because they were Navajo* that these individuals had contracted" (pp. 121-122) this acute respiratory distress syndrome.

Local doctors and health care workers were dumbfounded as to the underlying cause for the disease, and so the Centre for Disease Control (CDC) was called in to solve the mystery. The CDC carried out a series of laboratory tests that "failed to identify any of the deaths as caused by a known disease such as bubonic plague" (Centers for Disease Control and Prevention, 2012).

As additional testing continued, physicians and researchers repeatedly found that "The particular mixture of symptoms and clinical findings pointed… away from possible causes, such as exposure to a herbicide or a new type of influenza, and toward some type of virus". Tissue samples were analyzed by virologists at the CDC, ultimately leading to the identification of a previously undocumented type of hantavirus. The species of mouse (the deer mouse) known to carry and transmit this virus through its fecal matter and urine is not considered endemic to the Four Corners environment, and for this reason, hantavirus had not been considered in the original testing. Had hantavirus been included within the possible underlying diseases that were originally tested for, many of the deaths may have been prevented.

However, at least one of the healers in the affected Navajo reservations knew that a change in climate could result in the deer mouse being, at least temporarily, endemic to the Four Corners. In fact, early on in the investigation, a worker from the CDC , who was of Navajo descent, had gone to see one of the Navajo healers to ask about the disease. The healer replied by showing the CDC worker a photograph of a sand painting with a mouse in it, and he also told the worker that "many years ago such a sickness had occurred and that the sand painting had been used to treat it" (Arviso, & Cohen, 1999, p. 122). In reality, the sand painting did more than identify the particular breed of mouse responsible for the illness. It also explained why the population size of the deer mouse would increase: three or more years of excessive rain leads to increased production of the seeds of the dwarf pine trees in the area, and those seeds are one of the best food sources for the deer mouse. When finishing his sharing of the story of the sand painting, the healer told the worker to share this knowledge with the CDC, and more specifically, to "'Look to the mouse'" (p. 122).

Sadly, in all of the documented knowledge about this outbreak, there is no mention of anyone else (not even the Navajos living on the reservations) approaching the healer for information. Like the scientists, most Navajo people believed that this outbreak must be something new, beyond and foreign to their traditional knowledge. Upon receipt of the information from the Navajo healer, the CDC dismissed it because the deer mouse is not endemic to the four corners – they chose to ignore the context, namely the story of why the mice would come into the region.

The CDC has officially acknowledged that: "Navajo Indians… recognize a similar disease in their medical traditions, and actually associate its occurrence with mice. As strikingly, Navajo medical beliefs concur with public health recommendations for preventing the disease" (Centers for Disease Control and Prevention, 2012). Perhaps, if the traditional Navajo knowledge and ways of knowing had been valued by the scientific practitioners and, dishearteningly, by the Navajo people themselves, the hantavirus diagnosis would have occurred sooner, and fewer young and promising lives would have been lost. And, perhaps research into risk education would benefit from acknowledging the value of asking to learn and listening to traditional knowledges such as that of the Navajo healer -- knowledge which is neither rational (in the Western sense), abstract, or affective in origin.

**Tsunami, 2004**

On Dec. 26, 2004, a magnitude 9.0 earthquake, centered near the west coast of Sumatra and under the Indian Ocean, occurred. The energy released by this earthquake has been estimated to be equivalent to 23 000 atomic bombs equal to those used on Hiroshima. The resulting tsunami a few hours later had waves that moved at the speed of a jet (National Geographic News, January 5, 2005). Pictures and videos of homes, people, animals, and all kinds of belongings being swept into the ocean flooded the media. More than 150 000 people were instantly dead (or missing and presumed dead), with some estimates being placed at 250 000 and higher, and millions lost everything they had. Despite all of the technology and scientific models available to predict and communicate risks of earthquakes and tsunamis, scientists were unable to provide adequate warning for the event.

Yet, a number of Indigenous groups, frequently ignored or seen as inferior to other human inhabitants in the region, survived en masse. Unfortunately, the same was not true for the non-Indigenous people inhabiting the same islands and communities. It was a "survival determined by their in-depth knowledge of the environment" (Mercer, Dominey-Howes, Kelman, & Lloyd, 2007, p. 251). As an example, the Moken (or sea gypsies), an Indigenous group from Thailand who live on the Indian islands of Andaman and Nicobar "managed to anticipate the tsunami danger. Their knowledge of wind, tides, and the animals, which had been passed down from generation to generation, prepared them to deal with the natural disaster" (Perez, n. d., p. 1). Part of this knowledge included the silence of the cicadas, which was understood to tell the people to run for higher ground, and they did.

Similar examples from onslaught of the tsunami can be found throughout the region's Indigenous peoples, including those who live on Nias Island where, not only did the Indigenous people survive, but so too did their homes that were nearly 100 years old while the new modern homes on the island were destroyed. Neither is it emotional knowledge. It is the traditional knowledge of the people, the knowledge that has been preserved and carried forward through generations of oral traditions. It is spiritual knowledge, intuitional knowledge, physical knowledge, and probably even experiential knowledge. No known testing, isolation, compartmentalization, or abstraction of the knowledge was, or had been previously, done.

**Analysis of Risk Education, Risk Analysis, and Risk-Based Decision-Making; Navajo Plague, 1993; and Tsunami, 2004**

Based on the above understandings of risk education, analysis, and decision-making, along with the knowledge of the Navajo Plague and the 2004 tsunami, I will next proceed to my analysis of these risk-related topics. This analysis will again start with the discussion of the prominent features of these topics, followed by a discussion of the responses of the Traditional Western worldview and an Indigenous worldview to them, and then will end with the coding and explanation of concepts that have emerged.

**Prominent features of risk.**

Highlighted in the above discussion of risk and its positioning within mathematics curricula, I have chosen to focus in particular upon risk analysis, risk management, and risk-based decision-making. Risk management has become a feature of every day life, and thus decision-making in relation to risk has moved to the foreground of important content for students to learn. To date, within Saskatchewan, however, an analysis of the mathematics curricula reveals that risk is not explicitly included, and it is only as an indirect consequence of engaging with (objective and subjective) probabilistic knowledge that risk analysis and risk-based decision making might enter into a Saskatchewan student's education.

The assessment of risk is generally associated with two different ways of knowing about risk: personally defined and rationally derived; however there is great variance in terms of how individuals balance (or do not balance) the two. For example, Martignon and Krauss (2009) emphasize only the rationally derived ways of knowing and thinking about risk. In their work, the two researchers lay out a specific trajectory through which students learn how to rationally come to think about and assess risk.

Kent et. al. (2010), on the other hand, are researching the valuing of both personally determined interpretations of risk and rationally originating assessments of risk. In addition, they are investigating how to help individuals blend the two different ways of knowing about risk (personally and rationally) to make better-informed decisions.

The two specific risk-related examples (the Navajo Plague and the tsunami of 2004) provide a less theoretical look at how risk can be assessed and managed. In the case of the Navajo Plague, the traditional Navajo healer had knowledge related to the risk that was devastating his community, but others, were not valuing it such as many of the CDC researchers and the community members, because it was not rationally derived knowledge. However, the healer's knowledge was also not personally defined risk knowledge, as the healer himself had no previous personal experience with the disease that was over-running his community. Instead, the healer's knowledge was traditional cultural knowledge, based upon experiences and the knowledges of past healers and past down to him through the use of sand paintings. Because this way of knowing about the risk that faced his community was not valued within his community (or even sought) or by most of the CDC researchers, many people likely needlessly died from the onslaught of the hantavirus.

The example of the 2004 tsunami also highlights the existence of traditional cultural knowledge that is neither personally nor rationally derived. In this case, the Indigenous people chose to heed their traditional cultural knowledge resulting in their escaping from certain death. Other people in the same locations did not have, nor did they inquire about this traditional cultural knowledge, and consequently did not know about the preeminence of the tsunami. Most of them died as a result.

Thus, within the researching of the teaching and learning about risk analysis, risk management, and risk-based decision making, two ways of knowing are being considered as potentially valuable: personal response based upon one's own experiences and inferences, and rationally derived calculations of risk. From the two examples provided, however, a third way of knowing related to risk also emerges, namely through traditional cultural knowledges. With this understanding of how risk can be analyzed and understood, as well as an understanding of how risk is currently addressed within Saskatchewan's mathematics curricula, I now turn to the results of hermeneutically considering how the Traditional Western worldview would respond to these prominent features related to risk.

**Dialogue with the Traditional Western worldview.**

From the perspective of the Traditional Western worldview, the consideration of risk as a topic of learning for students would be considered beneficial from the perspective that the inclusion of risk within mathematics curricula would serve to further promote probabilistic thinking within the hierarchies of knowledges of value. Thus, knowing about risk through rational analysis involving probability would also be considered valuable.

Further, a person grounded within the Traditional Western worldview would value the research of Martignon and Krauss (2009) for its focus on discrete mathematical processes that provide rational knowledge pertaining to risk. However, considering personal impressions and inferences, such as what is emphasized in Kent et. al.'s (2010) research, would be seen as unnecessary and even muddying of the truth that rational analysis secures.

The final way of knowing about risk, which is introduced through the examples of the two risk-bound situations would be an even further step away from the ways of knowing that are valued within the Traditional Western worldview. The traditional cultural knowledge that provides information that is pertinent to understanding the risk in each situation is not rationally bound as is expected of the Traditional Western worldview. Instead, this knowledge would likely be perceived by a person grounded within the Traditional Western worldview as emotional or even spiritual knowledge, neither of which is considered of any value within this worldview.

Overall then, the Traditional Western worldview would only value risk being part of students' learning about probability, and in so doing, all of the emphasis would be upon objective (rational) knowledge versus knowledge based upon personal experience or "unverified" traditional cultural knowledge. With this understanding from risk and risk education from the perspective of the Traditional Western worldview, I next turn to a discussion of how a person grounded with an Indigenous worldview would respond to the same information about risk education.

**Dialogue with an Indigenous worldview.**

From the perspective of an Indigenous worldview, all three ways of knowing about risk (rational, personal, and traditional cultural) would be viewed as valuable as they would allow for diverse understanding of risk and risk-based decision-making. Further, the notion that in a given context, one or two of these ways of knowing might play a more important role in assessing and understanding the risk that is present than the other(s) would be equally acceptable.

Thus, from an Indigenous worldview perspective, in the cases of the Navajo plague and the 2004 tsunami, choosing to respond to the situation solely on the basis of traditional cultural knowledge would not have been questioned. Likewise, using any combination of the three ways of knowing discovered so far to make decisions and respond to the risks in those same questions would also be acceptable. Thus, a person grounded within an Indigenous worldview would accept the knowledge gained through Martignon and Krauss' (2009) research; however, the same person would see greater value in the work of Kent et. al. (2010) because of its emphasis on more than one way of knowing.

From the perspective of an Indigenous worldview, the incorporation of risk education into student learning would also be viewed as highly valuable. Since risk analysis, risk management, and risk-based decision-making are now a regular part of our daily lives, a person grounded within an Indigenous worldview would see such knowledge, if open to different ways of knowing, as knowledge of significant value because it would be contributing to the greater good.

With these understandings of how risk education and risk-based decision-making would be received by each of the two worldviews (the Traditional Western worldview and an Indigenous worldview), I can now turn to the final part of the analysis of these two areas. Returning to grounded theory, I now present and discuss the concepts that have emerged from both the data and my analysis of it so far.

**Coding and explanation.**

The discussion and analysis of risk education and risk-based decision making once again highlights the same concepts as previously mentioned. Moreover, the coding of these concepts against the data again supports the saturation of these concepts. A detailed description of how and where each of the concepts relates to risk education and risk-based decision making follows.

The concept of hierarchy between different kinds of knowledge and ways of knowing can be seen in the different approaches taken to researching risk education. Martignon and Krauss (2009) focus solely on students' familiarity and use of rational probability-related knowledge when considering questions of risk, placing it at the top of the hierarchy of ways of knowing. Kent, et. al. (2010), on the other hand, choose to value both rational knowledge and personal knowledge within their research. Further, they encourage the participants to determine for themselves the balance that they feel is appropriate between the two ways of knowing. In this way, Kent et. al. are challenging the notion of a hierarchy of ways of knowing, instead letting context and individuals define which, if either, of the two ways of knowing are more important.

The examples of the Navajo plague and the 2004 tsunami actually parallel these same approaches to hierarchy. In the case of the Navajo plague, no one challenges the supremacy of the rational approaches taken by the CDC researchers. Instead, the CDC officials dismiss the healer's traditional cultural knowledge because it does not align with their rational knowledge, and therefore conclude that the healer's knowledge is incorrect (only later to realize their error). The case of the 2004 tsunami, however, demonstrates how the Indigenous people's choice to trust their traditional cultural knowledge over the non-existent rational knowledge of others living in the same other, was in the end the best choice that could have been made. In one case, the hierarchy of knowledge denied consideration of alternate ways of knowing, while in the other case, there was no hierarchy of knowledge in play for some of the people (the Indigenous), while there may have been a hierarchy of knowledge of value for the non-Indigenous people.

Specialization is also common throughout the entire discussion of risk education and risk-based decision making. Within the research of Martignon and Krauss (2009), emphasis is placed upon students learning specialized probabilistic processes and knowledges in order to guide their decision-making in risk-related situations. The research of Kent et. al. (2010), on the other hand, seeks diversity in approaches to risk versus specialization. Notably, the two examples (the Navajo plague and the 2004 tsunami) return to specialized knowledge in the form of traditional cultural knowledge. In one case (the plague) the specialized (yet contextualized) knowledge is discredited and ignored, while in the other (the tsunami) the specialized (yet contextualized) knowledge is embraced by some. The result of these different responses to the specialized traditional cultural knowledges is ultimately one of life and death.

The singularity of knowledge is most clearly emphasized through the research of Martignon and Krauss (2009), although it can also be related to the tsunami of 2004. In the case of Martignon and Krauss' research, the approach that they take towards risk education emphasizes the singular importance (even dominance) of rational understandings of probability upon risk analysis and risk-based decision-making. Probability is presented the "right" way to analyze and make decisions related to risk. In the case of the tsunami of 2004, there is also a singularity of knowledge (specifically traditional cultural knowledge) presented; however, this may be more the result of the availability of data with respect to the event than a reflection of the kinds of knowledge that were available for use at the time.

Categorization and isolation of knowledge is also present within both Martignon and

Krauss' (2009) research and the research of Kent et. al (2010). In the first case, Martignon and Krauss isolate various pieces of mathematical knowledge to build towards (in a hierarchical fashion) students' singular understanding of risk. On the other hand, although they do categorize the ways of knowing risk into personal and rational, do not seek to have their participants keep these two ways of knowing isolated, rather they encourage the participants to find ways to integrate and bridge the personal and rational ways of knowing about risk. Even in my own analysis of the two situations (the Navajo plague and the 2004 tsunami), I have insisted on the categorization of the way of knowing being presented in order to differentiate it from the personal and the rational, which is emphasized in other research in the field.

Within Martignon and Krauss's (2009) research, relationship and context are only present to serve as tools for engaging the students in the desired probabilistic learnings. For Kent et. al. (2010), however, the context of Deborah's dilemma is deliberately chosen for how the participants may be able to relate to and personalize Deborah's situation. Further, the concepts of relationship and context play an even larger role within the two examples of risk-based situations (the Navajo plague and the 2004 tsunami) because the traditional cultural knowledge is contextual bound and it focuses on relationships between humans, animals, and the world that they live in.

The concept of power and authority emerges from this data and its analysis in two different ways. First, in Martignon and Krauss' (2009) research, the selection of a particular trajectory (and the accompanying justification for that specific trajectory) is clearly grounded with an assumption of both a hierarchy of probabilistic notions and knowledges as consequently within the authority and power attributed to each part of the trajectory. In the research of Kent et. al. (2010), rather than the power and authority being attributed by the researchers to a particular kind of knowledge, or a specific way of combining two different ways of knowing, the power and authority for making such decisions is turned over to the participants. In so saying, it must be noted that the researchers do nudge the participants into considering deeply where and how they will assign the power and authority of either or both ways of knowing and their resulting knowledges; however, they do not tell the participants any "right" way of assigning that power and authority. Finally, the two stories once again demonstrate both sides of the concept of power and authority. In the case of the Navajo plague, power and authority is assigned by (and to) the CDC officials (and by almost everyone else involved) specifically to scientific experimentation and analysis; whereas, in the case of the tsunami of 2004, power and authority was assigned to the traditional cultural knowledge, not by some outside specialist, but rather by the individuals who are directly in relation to the risk at hand.

Finally, the concept of abstraction is also a site of a dual interpretation within this section of the data. Whereas the research of Martignon and Krauss (2009) is focused on helping students obtain abstract understandings of a series of probability-related knowledges, Kent et. al. (2010) seek to find ways for their participants to find value within and grapple with their personal thinking about risk and risk-based decision-making.

At this point within my overall analyses, the concepts of hierarchy, specialization, singularity, categorization and isolation, relationship, power and authority, abstraction, and context (story) have consistently reappeared, and, I contend, the explanations of how these concepts related to the particular parts of data has resulted in a complete saturation of each of these concepts. Further, as each concept has been repetitively explored and explained through the contexts of the data being analyzed, all of the concepts have been crossing the artificial boundaries that the initial coding placed between them. A discussion of hierarchy, whether it is in support of the concept or in opposition, is naturally connected to specialization, singularity, and so one. In fact, all of the initial concepts that were coded and reported on by the end of my story's reflections upon my days as an undergraduate student have now also become saturated within a single conceptual category that I have labeled as "attributes of mathematics and the teaching and learning of mathematics".

As will be discussed shortly, this broad conceptual category next leads me to a new theory pertaining to my research question, "What ways of knowing and kinds of knowledge are, and possibly could be, valued within mathematics and the teaching and learning of mathematics?" Before I propose this new theory, however, I want to first return to the theoretical framework (of the Traditional Western worldview and an Indigenous worldview, which has played such a prominent (and hermeneutical) role within the analysis of my data. I do so to further justify the inclusion of this framework within my research and its use in the analysis by discussing how this framework might further be used within mathematics education research. I have chosen to place this discussion at this point within my dissertation, because I now believe that the reader has significant understanding of the two worldviews that will allow them to better understand how the framework might be used within research. Moreover, I have chosen this location, prior to the introduction of the theory that has emerged from the data and analysis, to further reinforce the readers appreciation for how the framework has contributed to the coding and saturation of the concepts and the conceptual category that emerged.

## Research Possibilities for my Theoretical Framework

As the theoretical framework that I presented at the end of my story has been foundational to the analysis of my research data, some final words in relation to the potential of the framework in relation to the researching of the teaching and learning of mathematics seem appropriate. Thus, in this section, I briefly describe how I see the framework of the Traditional Western worldview and an Indigenous worldview might be applied within future research.

First, I must acknowledge the obvious, which is that all of the analysis in my dissertation relating to the two worldviews has been a demonstration of one way that the framework can be used within the researching of mathematics and the teaching and learning of mathematics. In particular, I have been using the two worldview lenses to look into different aspects of mathematics and the teaching and learning of mathematics in order to identify the kinds of knowledge and ways of knowing that are being valued within different situations and presentations. I also propose that the worldview framework can be used to theorize about how the boundaries of the topic being researched would appear when grounded within each of the two worldviews or some mix of the two. In addition, my theoretical framework can also be used as a catalyst for change and the analysis of those changes.

In the following sections, I further elaborate upon each of these types of uses for the framework within research, providing examples for each coming from the results of a research project funded by the Dr. Stirling McDowell Foundation for Research into Teaching that was intended to investigate one question, but ended upon revealing and suggesting answers to very different questions (see Russell & Chernoff, 2013). I begin by considering the use of the two worldviews as lenses for looking in.

### Looking Inwards: Worldview Analysis of Mathematics and the Teaching and Learning of Mathematics

As previously noted, much of the analysis done so far in this dissertation falls into this particular way of using the theoretical framework in research. When using the worldviews to look inwards, the purpose is to look for connections and divergences between the subject matter (in this case, mathematics and the teaching and learning of mathematics) and each of the worldviews. The conclusions of such research could reveal not only a sense of alignment between the subject matter and the particular worldviews, but also a clear picture of what ways of knowing and kinds of knowledge are being valued and which are not. Such research is of value as it can be used to inform and give further research questions (as this very work demonstrates) by providing insights into the subject of the research that might not otherwise be obvious or even considered important regarding attitudes, beliefs, and processes.

As an example of this type of use of my theoretical framework (taken from the research project mentioned above) although the original research question was “what impacts does the grounding of the teaching and learning of mathematics within an Indigenous worldview have upon students’ (1) affective response to mathematics and mathematics learning, and (2) academic achievement in learning mathematics,” much of the data collected provided insights into the kinds of knowledge and ways of knowing that the teachers themselves were valuing in their teaching.  Although the aim of the research project was to have the teaching and learning of mathematics grounded within an Indigenous Worldview, there were numerous incidents in which the values held and communicated by the teachers could be more directly correlated to the values of the Traditional Western Worldview than to those of an Indigenous Worldview.  Reflections upon these conclusions has led me to consider the determining of one’s positioning with respect to the two worldviews prior to trying to incorporate any or all aspects of either or both is important for understanding what happens when changes are attempted.

**Examining and Challenging the Boundaries**

The second way in which the theoretical worldview framework can be used within research related to mathematics and the teaching and learning of mathematics is by first identifying and then challenging the boundaries placed upon the kinds of knowledge and the ways of knowing that are being valued.  The analyses done in previous sections of this dissertation also demonstrate the first part of this use of the framework in that they not was what was being valued identified, but also what was not.  By identifying limitations that can be observed through the two worldview lenses, the impacts of those limitations can then be researched.

Within the aforementioned research project (see Russell, & Chernoff, 2013), this type of use of my theoretical framework bore out when incidents were noted and analyzed where the teachers moved away from their attempt to be grounded within an Indigenous Worldview and back into the Traditional Western Worldview.  As an example, when confronted by her students being unable to complete a task involving the hundred chart, one of the teachers (Sharon) told myself (the researcher), and the other teachers in our working group that her only recourse was to reteach her students about the hundred chart despite her own misgivings about how the chart was constructed. At that moment, Sharon's openness to valuing diverse ways of knowing and kinds of knowledge was shut down by the event that had happened, and she was planning upon returning to the Traditional Western Worldview reasoning of their being one "right" way of obtaining and representing knowledge. Without this framework being front and centre within the research, this incident may very well have gone unnoticed or not critiqued; however, Sharon's statement immediately initiated a conversation, primarily amongst the teachers, about whether the hundred chart had to be given that particular layout, or if others were not only possible, but actually preferable.

Consequently, this discussion shifted Sharon's thinking out of the boundaries defined by the Traditional Western Worldview and back into thinking that was more aligned with an Indigenous Worldview. This is but one of many such examples of how shifts in the boundaries of the teachers thinking about the kinds of knowledge and ways of knowing that are of value were easily identified and ultimately challenged through the use of the worldview framework.

**A Catalyst for Change**

The final way in which the framework based upon the Traditional Western Worldview and an Indigenous Worldview can be used within research is as a catalyst for change. The two worldviews provide a schema for thinking about what ways of knowing and knowledge can, might, or even should be valued within mathematics and the teaching and learning of mathematics. Either worldview, or any portion thereof, can thus be used as a catalyst for desired change, the effectiveness of which can then be analyzed and reported on. Examples of how the worldview framework can be used in this capacity come in the after stories discussed by Russell and Chernoff (2013). In each of these after stories, the impact on the teachers and students of the teachers responses to incidents of intrusion by the Traditional Western Worldview into their teaching of mathematics are presented and discussed. In each instance, the teachers' awareness of what worldview they were grounding themselves in and why ultimately led to desired outcomes in the students learning. To this point in my dissertation, I have not applied my theoretical framework to my research in this final way; however, it will come to take a leading role in the theory that has emerged from the data and its analysis. I now move onto the presentation and discussion of that theory.

## The Emergence of a Theory

Through the saturation of the concepts coded and reported upon (namely, hierarchy, specialization, singularity, categorization and isolation, relationship, power and authority, abstraction, and context) within the analyses of the data, as well as the combining of these concepts into a saturated conceptual category ("attributes of mathematics and the teaching and learning of mathematics"), the emergence of a theory is now immanent. This theory is also directly tied to the claim that the theoretical framework used within all of the analyses may in fact hold a key to some different forms of resolution to the crises, difference, conflict, and confusion that has emerged within the data on mathematics and the teaching and learning of mathematics.

I have named the theory that I am about to describe the *Transreform Approach to Mathematics and the Teaching and Learning of Mathematics* (hereafter called the Transreform Approach). Stated plainly, the Transreform Approach is achieved by grounding oneself within an Indigenous Worldview. This explicitly means that the kinds of mathematical knowledge and ways of knowing that are valued (including how mathematics is taught and learned) align with the Indigenous Worldview. I have selected the term "transreform" with great purpose, in that the Transreform approach embodies all aspects of the prefix of "trans," referring not only to the joining of the traditional approach and the reform approaches to the teaching and learning of mathematics, but also to the notion of providing a means of transit between and beyond both of these assumed realities.

At first glance one might think this proposal illogical and detrimental, a proverbial swapping of the baby out for the bath water. Contrarily, I argue that by grounding ourselves within an Indigenous worldview, we would be ensuring that nothing related to mathematical knowledge and ways of knowing would be lost; rather, more (hopefully all) would be acknowledged and valued for how they contribute, particularly within specific contexts. This would be the case because, in abstract mathematical terms, the Traditional Western Worldview can be contained within an Indigenous Worldview. That is to say, because an Indigenous Worldview considers what knowledge and ways of knowing are of value based upon context, then the knowledge and ways of knowing valued within the Traditional Western Worldview, which have contexts in which they are not just needed, but mandatory, would have places in an Indigenous Worldview where they would be most valued. Conversely, the Traditional Western

Worldview would not be able to always house the values of an Indigenous Worldview.

It is also important to note that the Transreform Approach is not a defining of a middle ground within a dichotomized situation, such as seen in the math wars. Instead, the Transreform Approach removes the instances of dichotomy by allowing the two ends of any spectrum to become just two options within a far more extensive playing field of possibilities, options that will be more or less valued depending upon the place, time, and person (people) involved. In some cases, the context or place will dictate what kinds of knowledge and ways of knowing are most valued (for instance, in a scientific journal, abstract and isolated mathematics would be of most value), in other instances, the place in which it is occurring as well as the time available may decide what is most valuable. On other occasions it may be the person, who arbitrarily or with great purpose, decides what is of value.

**Potential of the Theory**

The Transreform Approach has many potential impacts (related specifically to the data presented within this dissertation), which I now state as a list of numbered hypotheses (with defense) below:

1. The Transreform Approach can help Indigenous students, in fact all students, find ways to relate to and "find themselves" within the mathematics they learn and use. By grounding teaching and learning within an Indigenous Worldview, opportunities for students to bring their ways of knowing and knowledge about mathematics forward will naturally occur, giving all students entry points into the learning, regardless of whether the ultimate mathematics to be learned is to be abstracted and categorized, or not. It will allow the students to bring to the table their stories, their emotions, their intuitions, their physical understandings, their spiritual understandings, and so on, in order to address the questions posed, and by doing so, personalize the mathematics for the individual students. This I believe is what was missing for the student that told me that he now "knew what mathematics wanted from [him]." That student had been looking for a way to build a relationship with mathematics, and by grounding the teaching and learning within an Indigenous Worldview, the creation and maintaining of such relationships can be possible in ways most meaningful to the individual students.
2. The Transreform Approach does not negate existing boundaries on mathematics

and the teaching and learning of mathematics; rather, it will clarify those boundaries with respect to their context, while extending and even creating new boundaries for mathematical knowledge and ways of knowing that are suited to alternative contexts and needs. Thus, the Transreform Approach does not throw out the Traditional Western worldview; rather, it recognizes it as part of a larger set of values. In this way, this theory is not a middle ground, nor is it a total remake, such as changing from traditional to reform approaches might appear, it is a broadening of perspective to make room for not only the traditional and the reform, but also for those ways of knowing and kinds of knowledge which have never been valued within Western mathematics. The Transreform Approach would also result in the reclaiming and re-envisioning of Bishop's term 'Mathematics', where absolute power and authority would not be given over to Western mathematics, but instead, the name would now stand for all of Mathematics – Western, enthnomathematical, and whatever else may come to pass. Western society only owns the word mathematics because it assumed the authority to claim it and the power of using it. The Transreform Approach gives us the opportunity to redefine mathematics (or Mathematics) and make it word associated with all mathematical ways of knowing and kinds of knowledge.

3. The Transreform Approach can expand and strengthen mathematical understanding and ability. By opening mathematics up to alternative ways of knowing and kinds of knowledge, such as the traditional knowledges foundational to the examples of the hantavirus outbreak and the tsunami, mathematics itself will have ways to grow previously denied to it, and all people (mathematicians and non-mathematicians, Indigenous and non-Indigenous) will benefit from the changes and expansion in knowledge and thinking.
4. The Transreform Approach can eliminate the need for the math wars. In some respects, how this would be achieved is similar to that proposed by those seeking the middle ground; however, the Transreform Approach would not require a singular resolution. That is to say, the Transreform Approach would not relegate one approach to be the servant of the other, such as reform methods serving the role of solidifying traditional Western mathematical knowledge. Within the

Transreform Approach, the traditional approach and the reform approach could interact in any number of ways, or even completely alternative approaches could enter into the processes of teaching and learning. For example, neither the traditionalists nor the reformists propose approaches that would be easily amenable to the inclusion of the traditional knowledge found in data. Consequently, the Transreform Approach could help guide curricular reform to be more meaningful and responsive to all students and their particular learning styles and needs.

5. The Transreform Approach would be supportive and receptive to the use of my theoretical worldview framework. The use of this framework has shown to provide unique, meaningful, and useful insights into mathematics and the teaching and learning of mathematics. In fact, without this theoretical framework, the theory I propose would not have come into existence.
6. The Transreform Approach broadens what Bishop (1991) defines as the "openness of mathematics" or what other philosophies and claims call the "universality of mathematics" so that mathematics communication and understanding becomes truly open. Within the Transreform Approach, there is not one way of communicating about mathematics because there is not one way of representing it. Thus, not only people who are Western mathematicians can communicate with each other, regardless of where they are from; anyone can communicate mathematically and meaningfully with anyone else.
7. Finally, while removing all of the points of contention and sources of tension from my story and the data presented, the Transreform Approach does require the conceptualization of a philosophy of mathematics that supports and substantiates the theory. I will now briefly outline such a philosophy.

**Philosophy of the Transreform Approach**

Since none of the existing philosophies of mathematics aligned with either the Traditional Western or an Indigenous worldview, then it is safe to assume that none of those same philosophies can embrace the knowledge and ways of knowing valued by the Transreform

Approach. And, if none of the existing philosophies are capable of embracing the Transreform Approach, then a new philosophy is needed (to use a Western "if-thenism"). It seems only fitting that this philosophy would acknowledge the circumstances under which it is created, and thus it should include the term "transreform" to represent how this philosophy must be broad enough, and open enough, to allow traditional and reform mathematics teaching and learning to be seen as valuable. Further, the term transreform, as associated previously with grounding of the teaching and learning of mathematics within an Indigenous worldview, communicates the valuing of all kinds of knowledge and ways of knowing mathematically (for the contexts in which they are most relevant), regardless of whether they can be housed within either camp from the math wars or neither of them.

Next, this philosophy has to be radical, but more than that, it needs to be radically humanistic. Scholarly research tells us that humanism, and recognition of the "human face of things" (such as mathematics, in this case), can in itself be a slippery slope towards oppression and subjugation. In particular, Smith (1999) explains how humanistic research methods actually gave permission to researchers to apply very specific definitions of who is human and who is not within their research, strengthening the impact of the hegemony and oppression that continues to grow as a result of colonization and slavery. The humanistic feature of this proposed philosophy must not allow for this kind of segregation and devaluing of certain people and ideas – it must be radically different, a radical humanism that equitably values all people, all of their knowledges, and all of their ways of knowing.

Marrying this understanding of radical humanism with the transreform grounding of mathematics results in a new philosophy of mathematics, which has room for both Bishops' Mathematics, and for all of the mathematics that has been disregarded by that Mathematics and its practitioners. It is the conscious grounding of what mathematics is in an Indigenous worldview; where dichotomies cease to exist and viability truly determines worth (and possibly truth); a contextual and time-defined variable whose measurement is solely the responsibility of those who are using it in a particular time and within a particular context. This is the *transreform radical humanistic philosophy of mathematics*.

Within this philosophy of mathematics, those who stand outside a particular context and time may only reflect, but not pass judgment on how others have proceeded; recognizing how the mathematics might have been different for them if they had been in that time and place, without condemning the mathematics that actually is or was.  Truth belongs to the mathematician(s) at work; it can inform other mathematicians in their work – it can contradict, complement, or just expand the work of others, but it can never be presumed to be the truth of others.

School mathematics, then, becomes mathematics in which students contemplate the truths of mathematical questions and activities, understanding other's truths and relating them to one's own, but ultimately developing the mathematical tools necessary for the students to succeed in their mathematical futures, whatever form and in whatever context they might be encountered.

Western mathematics belongs in school mathematics, as it does elsewhere; however, it must necessarily relinquish absolute authority when confronted by someone else's viable alternatives.  If mathematics can come to live as transreform radical humanistic mathematics, there is promise for great strides being made in reversing, and even eliminating, the oppressing and privileging of people and their thinking, as well as strengthening everyone's mathematical understandings and abilities.  There is also the promise of mathematics itself growing as a result. The philosophy of transreform radical humanistic mathematics proposes moving away from saying "it may be possible for other mathematics to exist" to saying "let's celebrate, and benefit from the diversity of what we know and can come to know as mathematics."

## Reflections on my Choice of Data Sources and Methodologies

At times, as I worked through the sharing and analysis of my data, I questioned whether my choices in terms of data sources, methodologies, and methods were ultimately being influenced by where my worldview positioning was located, and of course it was. Worldview influences all of our decisions, as that is its role. However, I also came to realize that by assuming the role of speaking with both the Traditional Western worldview and an Indigenous worldview, I was also allowing myself, and my research to be open to alternative perspectives that would not have been present, or even considered, without the use of this theoretical framework.

So, at this point, I provide, in a style grounded within the Traditional Western worldview (because of its categorization and isolation of features of my research, without consideration of relationships or story), my analysis of my data sources and methodologies choices:

**Table 2: Data Source/Methodology Worldview Alignments**

| | **Traditional Western worldview alignment** | **Indigenous worldview alignment** |
|---|---|---|
| **Data Sources** | | |
| My story | • Too much context<br>• Very little valid knowledge<br>• Justification of future data sources not necessary | • Foundational<br>• Establishing relationships |
| Philosophy of mathematics<br>Math Wars<br>Indigenous Students and Ethnomathematics,<br>Risk Education<br>Use in Research | • Factual data<br>• Too much consideration of story and relationships | • Relationship to story<br>• Contextualization<br>• Diversity of knowledge |
| **Methodologies** | | |
| Auto/ethnography | • Too much focus on self – distancing needed | • Self in relation to context and others is of value |
| Gadamerian hermeneutics | • Traditional Western worldview dialogue establishes what is important<br>• Indigenous worldview dialogue is impractical | • Traditional Western worldview dialogue is limiting<br>• Indigenous Worldview dialogue is accommodating<br>• Two dialogues together are respecting of dialogue |
| Grounded Theory | • Initial coding – detailed and categorized<br>• Axial coding – too much broadening of the categories<br>• Theory generation – not measurable nor are all data sources reproducible | • Initial coding – fairly narrow coding; however, diversity in codes<br>• Axial coding – establishment of relationships and contextualizing<br>• Theory generation – recognizing of valuable relationships and contributions of both worldviews |

What I hope that this table demonstrates is not an equality in alignment (Traditional Western worldview) nor an equitable alignment (Indigenous worldview); rather, I hope it shows that neither worldview was wholly denied within my research. For each worldview, there were sites of value; moreover, those sites of value all contributed to my overall analyses, coding, and theory. In recognizing this, although I still feel the pull towards more equality or equity at times, I am comfortable in my choices of data sources and methodologies, and the conclusions that they ultimately afforded me. Perhaps, the above table even embodies my own worldview in relation to mathematical knowledge and ways of knowing that are of value – a "Gadamerian-esque" dialogue between the two worldviews.

## Reflections

When reflecting upon the knowledge that the Indigenous people had and used to avoid the ravages of the tsunami, Mercer, Dominey-Howes, Kelman, and Lloyd (2007) stated that these kinds of knowledge and ways of knowing are "increasingly recognized in the international arena, yet [are] frequently overlooked in practice" (p. 247), often with great cost to human lives and wellbeing. It is time to stop just seeing the other possibilities but not valuing them, and the Transreform Approach provides a theoretical basis for how this can be done.

Mathematics and the teaching and learning of mathematics are rife with conflicts and crises – some well documented (such as the math wars and the struggles of Indigenous students) and others less well known (such as the variety of and clashing between the philosophies of mathematics). I propose, not without evidence, that my theoretical framework and the resulting Transreform Approach theory could be important pieces in the work to eliminate these problems as well as to expand what mathematics is and what should be valued within it.

The Transreform approach requires one's examination and reflection upon their own views about mathematics and the teaching and learning of mathematics, as well as the kinds of knowledge and ways of knowing that they do and do not value. The Transreform Approach gives a route for attaining the equity in mathematical thinking and doing that ethnomathematics demonstrates is sorely missing, and it is an avenue through which students and teachers can engage in culturally sensitive mathematics education.

The Transreform Approach provides the opportunity to change how "instutionalized Eurocentric curricula constantly reinforce the racial and sexual inferiority complexes among people of color and women" (Anderson, 1997, p.293), by acknowledging and valuing the knowledge and ways of knowing of everyone. As Barta (2001) so eloquently wrote, "Our classrooms must become places where children learn to value differences and respect variety. For hope to be alive, we must teach our children that as human beings, we do many of the same things, yet because of our individual and collective cultures, we do those things differently. The problem is not the differences but rather our learned responses to them. …. In our mathematics classrooms, we can help our students learn that we all count" (p. 305), and this can happen through the Transreform Approach.

Finally, the Transreform Approach challenges Van Eijick and Roth's (2007) claim that the Traditional Western worldview and an Indigenous worldview "are incommensurable with each other" (p. 935) by proposing that, instead of trying to force an Indigenous worldview into the authoritative stance of the Traditional Western worldview, one can instead invite the Traditional Western worldview into its own special and abstract location within an Indigenous worldview. From that positioning, the Traditional Western worldview can maintain its authority and power (within context), while an Indigenous worldview can continue to seek and value diversity in the kinds of knowledge and ways of knowing that it has.

## Moving Forward with the Transreform Approach

With the proposal of a new theory, the inevitable question of "so what?" arises – what can be done with the theory now that it's been proposed? Clearly, just issuing a mandate to all teachers, students, administrators, and parents, as well as the society as a whole, is not going to work, just as proposals to change how mathematics has been traditionally taught to approaches that actively engage the students in knowledge creation and understanding have not been widely (or in many cases, successfully) implemented and maintained. So, where to go from here?

First, it must be recognized that the assumption that mathematics is, almost en masse, being taught from the perspective of the Traditional Western worldview is not advisable, even though much of the data and it's analysis presented throughout this document may give that impression. To do so would be to make assumptions about the teachers and their worldviews. Even to assume that all teachers of Western origins are grounded within the Traditional Western worldview (see the general discussions of worldviews). Thus, it would seem the best place to start would be to investigate how the teaching and learning of mathematics in different classrooms aligns and or misaligns with each of the two worldviews. By determining what kinds of knowledge and ways of knowing are being valued within classrooms, wiser, and more respectful, decisions about how to implement the Transreform Approach can be made.

It will also be important to engage teachers (as well as students, parents, administration, the public) in discussion and reflections upon what the grounding of the teaching and learning of mathematics within an Indigenous worldview could look like. It will be important to remember that the Transreform Approach may require fundamental belief (worldview) changes for many people, and such changes take lots of experiences, time, and support.

Only once these first two endeavours are in place, can one actually start to research, in earnest, the Transreform Approach in action. Moreover, such research will require patience and time, as teachers, students, parents, administrators, and the public, in general, come to both understand, and ultimately implement this new theory. However, having said that, it should also be noted that the time is rife with events and situations that will help with the acceptance of an Indigenous worldview. As scientists turn to Indigenous knowledges seeking ideas for how to deal with climate change, as reconciliation for the damage (including genocide) done by Canada's residential schools upon our Indigenous peoples is being called for and supported, and as the Métis and all First Nations peoples are being extended the rights previously only given to status Indians, Canadians (and thus Saskatchewanians) are starting to consider additional ways of thinking and knowing different from those that they were taught and valued. The Transreform Approach could thus be part of thus societal shift, but it needs to be done with respect for everyone, and all worldviews involved.

References

Adams, T. E., Holman Jones, S., & Ellis, C. (2015). *Autoethnography: Understanding Qualitative Research*. [Kindle Edition]. Retrieved from http://www.amazon.ca.

Aitken, N. E., & Bruised Head, A. (2008). Native reserve students' and native public school students' ways of knowing math. *The Canadian Journal of Native Studies, XXVIII*(2), 295–312.

Allen, N. J., & Crawley, F. E. (1998). Voices from the bridge: Worldview conflicts of Kickapoo students of science. *Journal of Research in Science Teaching, 35*(2), 111-132.

Anderson, S. E. (1997). Worldmath curriculum: Fighting eurocentrism in mathematics. In Powell & Frankenstein (Eds.), *Ethnomathematics: Challenging eurocentrism in mathematics education.* (pp. 291- 306). Albany, NY: State University of New York Press.

Arviso Alvord, Dr. L., & Cohen Van Peet, E. (1999). *The Scalpel and the Silver Bear.* Toronto, ON: Bantam Books.

Barnhart, R., & Kawagley, A. O. (2005). Indigenous Knowledge Systems and Alaska Native Ways of Knowing. *Anthropology & Education Quarterly, 36*(1), 8 – 23.

Barta, J. (2001). Mathematics and Culture. *Teaching children mathematics*. 7(60): 305.

Barton, A. C., & Darkside. (2005). Greater objectivity through local knowledge. In W. –M. Roth (Ed.), *Auto/Biography and Auto/Ethnography: Praxis of Research Method* (pp. 23 – 47). Rotterdam, NL: Sense Publishers.

Barton, B. (2009). *The language of mathematics: Telling mathematical tales*. New York: Springer.

Barton, B., Fairhall, U., & Trinick, T. (1998). Tikanga Reo Tātai: issues in the development of a Mäori mathematics register. *For the Learning of Mathematics, 18*(1), 3–9.

Barrow, J. D. (2000). *The Book of Nothing.* London, England: Random House UK, Limited.

Barthold, L. S. (n.d.). Hans-Georg Gadamer (1900 – 2002). *Internet Encyclopedia of Philosophy.* Online: http://www.iep.utm.edu/gadamer/#SH3b.

Battiste, M. (2002). *Indigenous Knowledge and Pedagogy in First Nations Education: A Literature Review with Recommendations.* Prepared for the National Working Group on Education and the Minister of Indian Affairs, Indian and Northern Affairs Canada

(INAC). Ottawa, ON. Online: http://www.afn.ca/uploads/files/education/24._2002_oct_marie_battiste_indigenousknowledgeandpedagogy_lit_review_for_min_working_group.pdf

Becker, J. P, & Jacob, B. (1998). Math war developments in the United States (California), *ICMI Bulletin No. 44*. Online: http://lsc-net.terc.edu/do/paper/8090/show.1.html.

Bishop, A. J. (1991). *Mathematical Enculturation: A Cultural Perspective on Mathematics Education.* Dordrecht, NL: Kluwer Academic Publishers.

Boaler, J. (2015). *What's Math Got to Do with It? How Teachers and Parents can Tranform Mathematics Learning and Inspire Success*. Penguin Publishing Group. [Kindle Edition]. Retrieved from http://www.amazon.ca.

Borovcnik, M., & Kapadia, R. (2011). Determinants of decision-making in risky situations. *International Statistical Institute: Proceedings of the 58th World Statistical Congress (pp. 5503- 5508),* Dublin, Ireland.

Breuer, F. (2005). Scientific experience and the researcher's body. In W. –M. Roth (Ed.), *Auto/Biography and Auto/Ethnography: Praxis of Research Method* (pp. 99 – 118). Rotterdam, NL: Sense Publishers.

Butterworth, B. (1999). *The Mathematical Brain.* London, GB: MacMillan Publishers Ltd.

Canadian Metis Council (n.d.). Online: http://www.canadianmetis.com/Qualifying.htm.

CBCNEWS (2007). *Math curriculum changes don't add up: Experts*. Online: http://www.cbc.ca/news/canada/newfoundland-labrador/story/2007/04/05/math-changes.html

Centers for Disease Control and Prevention (2015). *Tracking a Mystery Disease: The Detailed Story of Hantavirus Pulmonary Syndrome (HPS).* Online: http://www.cdc.gov/hantavirus/hps/history.html.

Charmaz, K. (2012). The power and potential of grounded theory. *Medical Sociology Online, 6*(3), 2 – 15.

Cheek, H. N. (1984). A suggested research map for Native American mathematics education. *Journal of American Indian Education, 23*(2), 1–9.

Coltman, R. (1998). *The Language of Hermeneutics: Gadamer and Heidegger in Dialogue.* Albany, NY: State University of New York Press.

Connell, D. D. (Executive Producer). (1970). *Sesame Street* [Television Series]. New York, NY: PBS.

Corbin, J., & Strauss, A. (1990). Grounded theory research: Procedures, canons, and evaluative criteria. *Qualitative Sociology, 13*(1), pp. 3-21.

Creath, R. (Spring, 2013). Logical Empiricism. In Edward N. Zalta (Ed.), *The Stanford Encyclopedia of Philosophy (Spring 2013 Edition),* Online: http://plato.stanford.edu/archives/spr2013/entries/logical-empiricism.

Crosswhite, F. J. (1985). *Second International Mathematics Study: Summary Report for the United States*. Washington, D.C.: National Center for Education Statistics.

Davison, D. M., & Mitchell, J. E. (2008). How is mathematics education philosophy reflected in the math wars? *The Montana Mathematics Enthusiast, 5*(1), 143–154.

Denzin, N. K. (2014). *Interpretive Autoethnography (2nd Edition)* [Kindle Edition]. Sage Publications. Retrieved from http://www.amazon.ca

Denzin, N. K., & Lincoln, Y. S. (2011). *The SAGE Handbook of Qualitative Research, 4th Edition.* Sage: Thousand Oaks, CA.

Devlin, K. (2000). *The Math Gene: How Mathematical Thinking Evolved and Why Numbers are Like Gossip.* New York, NY: Basic Books.

Dostal, R. J. (2002a). Gadamer: The man and his work. In R. J. Dostal (Ed.) ,*The Cambridge Companion to Gadamer* (pp. 13 – 35). Cambridge, UK: Cambridge University Press.

Dostal, R. J. (2002b). Introduction. In R. J. Dostal (Ed.), *The Cambridge Companion to Gadamer* (pp. 1 – 12). Cambridge, UK: Cambridge University Press.

Ellis, C., Adams, T. E., Bochner, A. P. (2011). Autoethnography: An overview. *Forum: Qualitative Social Research, 12*(1). Online: http://www.qualitative-research.net/index.php/fqs/article/view/1589/3096.

Ermine, W. (1995). Aboriginal epistemology. In M. Battiste & J. Barman (Eds.), Fi*rst Nations education in Canada: The circle unfolds* (pp. 101–112). Vancouver: UBC Press.

Ernest, P. (1997). The legacy of Lakatos: Reconceptualising the philosophy of mathematics. *Philosophia Mathematica, 3*(5), 116-134.

Ernest, P. (1991). *The Philosophy of Mathematics Education.* New York, NY: The Falmer Press.

Eytemology Online (2015). Online: www.etymonline.com.

Flick, U. (2014). *An Introduction to Qualitative Research*. [Kindle Edition.] SAGE Publications.

Retrieved from http://www.amazon.ca

Gadamer, H. G. (1989). *Truth and Method* (J. Weinsheimer, & D. G. Marshall, Trans). New York, NY: The Crossroad Publishing Company.

Glaser, B., G., & Holton, J. (2004). Remodeling grounded theory. *Forum: Qualitative Social ResearchSozialforschung, 5*(2), Article 4. Online: http://www.qualitative-research.net/index.php/fqs/article/view/607

Glaser, B. G., & Strauss, A. L. (1967). *The Discovery of Grounded Theory: Strategies for Qualitative Research.* New Brunswick, USA: Aldine Transaction. [Kindle Edition.] Retrieved from http://www.amazon.ca

Graham, B. (1988). Mathematical education and Aboriginal children. *Educational Studies in Mathematics, 19*(2), 119–135.

Greer, B., Mukhopadhyay, S., Powell, A. B., & Nelson-Barber, S. (2009). *Culturally Responsive Mathematics Education.* New York, NY: Routledge.

Grondin, J. (2002). Gadamer's basic understanding of understanding . In R. J. Dostal (Ed.), *The Cambridge Companion to Gadamer* (pp. 36 – 51). Cambridge, UK: Cambridge University Press.

Heath, H., & Cowley, S. (2004). Developing a grounded theory approach: A comparison of Glaser and Strauss. *International Journal of Nursing Studies, 41,* 141-150.

Henderson, J. Y. (2000). Postcolonial ghost dancing: Diagnosing European colonialism. In M. Battise (Ed.) ,*Reclaiming Indigenous Voice and Vision* (57-76). Vancouver, BC: UBC Press.

Hersch, R. (1997). *What is Mathematics, Really?* New York, NY: Oxford University Press.

Howard, P., & Perry, B. (2005). Learning mathematics: perspectives of Australian Aboriginal children and their teachers. In H.L. Chick & J. L. Vincent (Eds), *Proceedings of the 29th Conference of the International Group for the Psychology of Mathematics Education, Vol. III* (pp. 153–160). University of Melbourne: Psychology of Mathematics Education.

Irzik, G., & Nola, R. (2007). Worldviews and their relation to science. *Science & Education, 18*(6-7), 729-745.

Kaplan, R. (1999). *The Nothing that is: A Natural History of Zero.* New York, NY: Oxford University Press.

Kawasaki, K. (2006). Towards worldview education beyond language-culture incommensurability. *International Journal of Science and Mathematics Education, 5,* 29-48.

Kent, P., Pratt., D., Levinson, R., Yogui, C., & Kapadia, R. (2010). Teaching uncertainty and risk in mathematics and science. In C. Reading (Ed.), *Data and context in statistics education: Towards an evidence-based society. Proceedings of the Eighth International Conference on Teaching Statistics (ICOTS8)*, Ljubljana, Slovenia.

Kitcher, P., & Aspray, W. (1988). An opinionated introduction. In W. Aspray, & P. Kitcher (Eds.), *History and Philosophy of Modern Mathematics,* (pp. 3 – 57). Minneapolis, MN: University of Minnesota Press.

Kovach, M. (2009). *Indigenous Methodologies: Characteristics, Conversations, and Contexts.* Toronto, ON: University of Toronto Press.

Lakatos, I. (1976). *Proofs and refutations.* Cambridge: Cambridge University Press.

Lakatos, I. (1978). A renaissance of empiricism in the recent philosophy of mathematics. In J. Worral & G Currie (Eds.), *Mathematics, Science and Epistemology: Philosophical Papers Vol 2,* (pp. 24 - 42.) Cambridge University Press: Cambridge, MA.

Lakoff, G., & Núñez, R. E. (2000). *Where Mathematics Comes From: How the Embodied Mind Brings Mathematics into Being.* New York, NY: Basic Books.

Little Bear, L. (2000). Jagged worldviews colliding. In M. Battiste (Ed.), *Reclaiming Indigenous Voice and Vision,* (pp.77-85). Vancouver, BC: UBC Press.

Loewenberg Ball, D., Ferrini-Mundy, J., Kilpatrick, J., Milgram, R. J, Schmid, W., & Scharr, R. (2005). Reaching for common ground in K-12 mathematics education. *Notices of the AMS.* Online: http://www.ams.org/notices/200509/comm-schmid.pdf.

Lunney Borden, L., & Wagner, D. (2007). 'Show me your math': Inviting children to do ethnomathematics. *Proceedings of the 29th annual meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education, Lake Tahoe, USA, October, 2007.*

Lydon, C., & Kapalan R. (Speakers). (2000, August 8). *The History of Zero.* The Connection [Online radio]. WBUR (Producer). http://archives.wbur.org/theconnection/2000/08/30/the-history-of-zero.html.

Lysyk, B. (2012). *2012 Report – Volume 1.* Online: https://auditor.sk.ca/pub/publications/public_reports/2012/Volume_1/2012v1_fr.pdf.

MacBeth, D. (2001). Naturalism in the philosophy of mathematics, in C. M. Mi and R. Chen (Eds.), *Naturalized Epistemology and Philosophy of Science, Rodopi Philosophical Studies 7* (pp. 87-103). Rodopi: Amsterdam. Online: http://academia.edu/790925/Naturalism_in_the_Philosophy_of_Mathematics.

Malcolm, C., Sutherland, D., Keane, M. (2008). Forum: Teaching IK in school science: depths of understanding, nuances, and just do it. *Cultural Studies of Science Education, 3,* 614-621.

Martignon, L., & Krauss, S. (2009). Hands-on activities for fourth graders: A tool box for decision-making and reckoning with risk. *International Electronic Journal of Mathematics Education, 4*(5), 227-258.

Mather, J. R. C. (1997). How do American Indian fifth and sixth graders perceive mathematics and the mathematics classroom? *Journal of American Indian Education, 36*(2), 9–18.

McCarthy, D. (Producer). (1970). *Mr. Dressup* [Television Series]. Toronto: CBC.

McCarthy D. (Producer). (1970). *The Friendly Giant* [Television Series] . Toronto: CBC.

Mercer, J., Dominey-Howes, D., Kelman, I., & Lloyd, K. (2007). The potential for combining indigenous and western knowledge in reducing vulnerability to environmental hazards in small island developing states. *Environmental Hazards: Human and Policy Dimensions, 7*, 245-256.

Merriam-Webster (2015). *Definition of BRICOLAGE*. Online: http://www.merriam-webster.com/dictionary/bricolage.

Meyer, M. A. (1998). Native Hawaiian Epistemology: Sites of Empowerment and Resistance. *Equity and Excellence in Education, 31*(1), 22-28.

Meyer, M. A. (2003a). Hawaiian Hermeneutics and the Triangulation of Meaning: Gross, Subtle, Casual. *Canadian Journal of Native Education, 27*(2), 249-255.

Meyer, M. A. (2003b). *Ho'oulu: Our time of becoming*. Honolulu, Hawai'i: 'Ai Pōhaku Press.

Michell, H. (2005). Nēhîthâwâk of Reindeer Lake, Canada: Worldview, epistemology and relationships with the natural world. *The Australian Journal of Indigenous Education, 34*, 33-43.

Moslehian, M. S (2004). Postmodern view of humanistic mathematics. *Philosophy of Mathematics Education, 18,* 1-5. Online: http://people.exeter.ac.uk/PErnest/pome18/pdf/moslehian.pdf.

Moules, N. J. (2002). Hermeneutic inquiry: Paying heed to history and Hermes, an ancestral, substantive and methodological tale. *International Journal of Qualitative Methods, 1*(3), 1 – 21.

National Geographic News (January 7, 2005). *The Deadliest Tsunami in History?* Online: http://news.nationalgeographic.com/news/2004/12/1227_041226_tsunami.html.

NCTM. (1980). An Agenda for Action. Reston, VA: NCTM.

NCTM. (1989). Principles and standards for school mathematics. Reston, VA: NCTM.

Neel, K., & Fettes, M. (2010). Teaching numeracy in a community context: The roles of culture and imagination . *vinculum: Journal of the Saskatchewan Mathematics Teachers' Society, 2*(2), 45–63.

Neuschwander, C. (1998). *Amanda Bean's Amazing Dream.* New York, NY: Scholastic Press.

Nisbett, R. E. (2003). *The Geography of Thought: How Asians and Westerners Think Differently… and Why*. New York, NY: Free Press.

O'Brien, T. C. (1999). Parrot Math. *The Phi Delta Kappan, 80*(6), 434–438.

Pereira, L., Settelmaier, E., & Taylor, P. C. (2005). Fictive imagining and moral purpose: Auto/biographical research as/for transformative development. In W. –M. Roth (Ed.), *Auto/Biography and Auto/Ethnography: Praxis of Research Method,* (pp. 23 – 47). Rotterdam, NL: Sense Publishers.

Perez, F. Y. L. (n.d.) *Survival Tactics of Indigenous Peoples.* Online: http://academic.evergreen.edu/g/grossmaz/LEEPERFY/.

Poirier, L. (2007). Teaching mathematics and the Inuit community. *Canadian Journal of Math, Science & Technology Education*, *7*(1), 53-67.

Powell, A. B., & Frankenstein, M. (1997a). Ethnomathematical Knowledge. In Powell, A. B., & Frankenstein, M. (Eds), *Ethnomathematics: Challenging Eurocentrism in mathematics education* (pp. 5-11). Albany, NY: State University of New York Press.

Powell, A. B., & Frankenstein, M. (1997b). *Ethnomathematics: Challenging Eurocentrism in mathematics education.* New York: State University of New York Press.

Powell, A. B., & Frankenstein, M. (1997c). Reconsidering What Counts as Mathematical Knowledge. In A. B. Powell, & M. Frankenstein (Eds.), *Ethnomathematics: Challenging Eurocentrism in mathematics education* (pp. 193-200). Albany, NY: State University of New York Press.

Quine, W. V. (1981). Things and their places in theories. In W. V. Quine (Ed.), *Theories and Things* (pp. 1–23). Cambridge, MA: Harvard University Press.

Reed-Danahay, D. (2009). Anthropologists, education, and autoethography. *Reviews in Anthropology, 38*(1), 28 – 47.

Restivo, S., & Sloan, D. (2007). The sturm und drang of mathematics: Casualties, consequences, and contingencies in the math wars. *Philosophy of Mathematics Education Journal*, 20. Retrieved from: http://people.exeter.ac.uk/PErnest/pome20/.

Reys, R. E. (2001). Curricular controversy in the math wars: a battle without winners. *Phi Delta Kappan, 83*(3), 255–258.

Rodrigues, A. J. (2005). Unraveling the allure of auto/biographies. In W. –M. Roth (Ed.), *Auto/Biography and Auto/Ethnography: Praxis of Research Method* (pp. 119 – 130). Rotterdam, NL: Sense Publishers.

Roth, W. –M. (2005a). Auto/biography and auto/ethnography: Finding the generalized other in the self. In W. –M. Roth (Ed.), *Auto/Biography and Auto/Ethnography: Praxis of Research Method* (pp. 3 – 16). Rotterdam, NL: Sense Publishers.

Roy, H., & Morgan, M. J. (2008). Indigenous languages and research universities: Reconciling world views and ideologies. *Canadian Journal of Native Education, 31*(1), 232-247.

Russell, G. L., & Chernoff, E. J. (2013). Incidents of Intrusion: Disruptions of Mathematics Teaching and Learning by the Traditional Western Worldview. *Proceedings of the 35th Annual Meeting of the North American Chapter of the International Group for the Psychology of Mathematics Education.* Chicago, IL, USA.

Saskatchewan Ministry of Education (2007). *Grade 4 Mathematics Curriculum.* Regina, SK: Saskatchewan Ministry of Education. Online: https://www.edonline.sk.ca/bbcswebdav/library/curricula/English/Mathematics/Mathematics_4_2007.pdf

Saskatchewan Ministry of Education (2008). *Saskatchewan education indicators report: Pre-kindergarten to grade 12.* Regina, SK: Saskatchewan Ministry of Education. Online: http://www.education. gov.sk.ca/Default.aspx?DN=69222f44-c385-49d7-aff4-9dc770f47750.

Saskatchewan Ministry of Education (2009a). *Mathematic 9.* Regina, SK: Saskatchewan Ministry of Education. Online:

https://www.edonline.sk.ca/bbcswebdav/library/curricula/English/Mathematics/Mathematics_9_2009.pdf

Saskatchewan Ministry of Education (2009b). *Saskatchewan education indicators report: Pre-kindergarten to grade 12* . Regina, SK: Saskatchewan Ministry of Education. Retrieved from: http://www.education. gov.sk.ca/Default.aspx?DN=dfaff52e-a0f2-485e-9213-daaa59424ffe.

Saskatchewan Ministry of Education (2010). *2010 Saskatchewan Education Indicators Report: Pre-kindergarten to Grade 12.* Regina, SK: Saskatchewan Ministry of Education.

Saskatchewan Ministry of Education (2013a). *Foundations of Mathematics 30*. Regina: Saskatchewan Ministry of Education. Online: https://www.edonline.sk.ca/bbcswebdav/library/curricula/English/Mathematics/Mathematics_Foundations_Of_30_2012.pdf.

Saskatchewan Ministry of Education (2013b). *Workplace and Apprenticeship Mathematics 30.* Regina: Saskatchewan Ministry of Education. Online: https://www.edonline.sk.ca/bbcswebdav/library/curricula/English/Mathematics/Mathematics_Workplace_Apprenticeship_30_2012.pdf.

Sawaka, B. (1970). *Kingo Bingo* [Television Series]. Calgary.

Schoen, H. L., Fey, J. T., Hirsch, C. R., & Coxford, A. F. (1999). Issues and options in the math wars. *Phi Delta Kappan, 80*(6), 444–453.

Schoenfeld, A. H. (2004). The math wars. *Educational Policy, 18*(1), 253–286.

Scott, P. B. (1983). Mathematics achievement test scores of American Indian and Anglo students: a comparison. *Journal of American Indian Education, 22*(3), 17–19.

Seife, C. (2000). *Zero: The Biography of a Dangerous Idea.* Toronto, ON: Penguin Books.

Schelbert, L. (2003). Pathways of human understanding: An inquiry into Western and North American Indian worldview structures. *American Indian Culture and Research Journal, 27*(1), 61-75.

Silverman, H. J. (1991). *Gadamer and Hermeneutics.* New York, NY: Routledge.

Smith, L. T (1999). Decolonizing Methodologies: Research and Indigenous Peoples. New York, NY: Zed Books Ltd.

Snively, G., & Corsiglia, J. (2001). Discovering Indigenous science: Implications for science education. *Science Education, 85*(1), 6-34.

Stemhagen, K. (2007). Empiricism, contingency and evolutionary metaphors: getting beyond the math wars. *International Electronic Journal of Mathematics Education, 2(*2), 91–105.

Sternberg, G., Barrett, L., Blood, N., Glanfield, F., Lunney Borden, L., McDonnell, T., Nicol, C., & Weston, H. (2010). To become wise to the world around us: Multiple perspectives of relating Indigenous knowledges and mathematics education. *vinculum: Journal of the Saskatchewan Mathematics Teachers' Society,* pp. 7-19.

Sternberg, G., & McDonnell, T. (2010). Indigenous, personal and western mathematics: Learning from place. *vinculum: Journal of the Saskatchewan Mathematics Teachers' Society, 2(*2), 10–22.

Strauss, A., & Corbin, J. (1998). Grounded theory methodology: An overview. In N. K. Denzin, & Y. S. Lincoln (Eds.), *Strategies of Qualitative Inquiry,* (pp. 158- 183). Thousand Oaks, CA: SAGE Publications, Inc.

Strega, S. (2005). The view from the poststructural margins: Epistemology and methodology reconsidered. In L. Brown, & S. Strega (Eds.), *Research as Resistance: Critical, Indigenous, & Anti-oppressive Approaches,* (pp. 199-235). Toronto, ON: Critical Scholar's Press/Women's Press.

The Association For Qualitative Research (2015). *Bricolage.* Online: http://www.merriam-webster.com/dictionary/bricolage.

Trent, J. H., & Gilman, R. A. (1985). Math achievement of Native Americans in Nevada. *Journal of American Indian Education, 24*(1), 39–45.

U.S. Commission on Civil Rights (2003). *A quiet crisis: Federal funding and unmet needs in Indian country.* Online: http://www.usccr.gov/pubs/na0703/na0204.pdf.

Van Eijck, M., & Roth, W. (2007). Keeping the local local: Recalibrating the status of science and traditional ecological knowledge (TEK) in education. *Science Education, 91*(6), pp. 926-947.

Wachterhauser, B. (2002). Getting it right: Relativism, realism, and truth . In R. J. Dostal (Ed.), *The Cambridge Companion to Gadamer,* (pp. 52 – 78). Cambridge, UK: Cambridge University Press.

Wallis, C. (2006). How to end the math wars. *Time, 168*(2). Online: http://www.time.com/time/magazine/article/0,9171,1561144,00.html

Warnke, G. (2002). Hermeneutics, ethics, and politics. In R. J. Dostal (Ed., *The Cambridge Companion to Gadamer,* (pp.79 – 101). Cambridge, UK: Cambridge University Press.

WCP (1996). The Common Curriculum Framework for 10-12 Mathematics. Edmonton, AB: WCP. Online: https://www.wncp.ca/media/39471/frmwrkbycrs.pdf.

WiseGEEK (2016). *What is Bricolage?* Online: http://www.wisegeek.com/what-is-bricolage.htm.

Wisemath. http://wisemath.org

WNCP. (2006). The Common Curriculum Framework for K-9 Mathematics. Edmonton, AB: WNCP. Online: https://www.wncp.ca/media/38765/ccfkto9.pdf.

WNCP. (2008). The Common Curriculum Framework for 10-12 Mathematics. Edmonton, AB: WNCP. Online: https://www.wncp.ca/media/38771/math10to12.pdf.

Wynn, K. (1992). Addition and subtraction by human infants. *Nature, 358, 749-750.*